PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(First Semester)

Branch – STATISTICS

MATHEMATICS - I

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)
Answer ALL questions
ALL questions carry **EQUAL** marks   (10 x 1 = 10)

1   By fundamental theorem of algebra, $f(x) = p_0 x^n + p_1 x^{n-1} + \ldots + p_{n-1} x + p_n = 0$ where $p_0, p_1, \ldots, p_n$ are real and $p_0 \neq 0$ has _____ root.
  (i) n   (ii) n-1   (iii) atleast one   (iv) many

2   Form a rational cubic equation which has roots $2, 3, + \sqrt{-2}$.
  (i) $x^3 + 4x^2 - 21 = 0$ (ii) $x^3 - 8x^2 + 23x - 22 = 0$
  (iii) $x^3 - 4 = 0$ (iv) $x^3 - 23x = 0$

3   In m x n matrix if m = n then it is a _____ matrix.
  (i) scalar   (ii) diagonal   (iii) symmetric   (iv) square

4   Any 1 x n matrix is called _____ matrix.
  (i) unit   (ii) column   (iii) row   (iv) identity

5   $\dfrac{d}{dx}\left(\dfrac{x^3}{3x-2}\right) =$

  (i) $\dfrac{6x}{(3x-2)}$ (ii) $\dfrac{x^2(x+1)}{(3x-2)}$ (iii) $\dfrac{6x^2(x-1)}{(3x-2)^2}$ (iv) $\dfrac{3x}{(3x-2)^2}$

6   $\dfrac{d}{dx}(\sin^2 2x) =$
  (i) $2 \sin 4x$ (ii) $2 \cos 4x$ (iii) $2 \cos 2x$ (iv) $2 \sin 2x$

7   $\dfrac{d^n}{dx^n}(e^{ax}) =$

  (i) $a\, e^{ax}$ (ii) $a^n e^{ax}$ (iii) $\dfrac{e^{ax}}{a}$ (iv) $\dfrac{e^{ax}}{a^n}$

8   $\dfrac{d^4}{dx^4}(e^{-x}\cos x) =$
  (i) y   (ii) 4y
  (iii) -4y   (iv) $-y\, e^{-x}$

9   $\int 3x^2 dx =$ _____ $+ c$
  (i) $3x^3$ (ii) $6x^3$
  (iii) x^3 (iv) $6x$

10   $\int xe^x dx =$ _____ $+ c$
  (i) e^x (ii) $e^x(x-1)$
  (iii) xe^x (iv) $\sqrt{x}e^x$

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks   (5 x 5 = 25)

11 a   One of the roots of the equation $3x^5 - 4x^4 - 42x^3 + 56x^2 + 27x - 36 = 0$ is $\sqrt{5} + \sqrt{2}$. Find the other roots.

OR

   b   Solve $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in A.P.

12 a   If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, find $A^3 - 3A^2 - A + 9I$.

OR

   b   Express $\begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrices.

13 a   If $x^y = e^{x-y}$ find dy/dx.

OR

   b   If $y = \sqrt{(\sin x + \sqrt{\sin x + \sqrt{\sin x} \ldots\ldots \text{to } \infty}}$ find dy/dx.

14 a   If $y = (x + \sqrt{1+x^2})^m$ find $(1+x^2)y_2 + xy_1$.

OR

   b   If $f = (x^2 + y^2 + z^2)^{-1/2}$ find $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial y^2}$.

15 a   Evaluate $\int \dfrac{x^2 + 2x - 1}{\sqrt{x}} dx$.

OR

   b   Evaluate $\int \cos^4 2x\, dx$.

SECTION -C (40 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks   (5 x 8 = 40)

16 a   If the roots of $x^3 + px^2 + qx + r = 0$ are α, β, γ find

   (i) $\sum \alpha^2 \beta$ (ii) $\sum \alpha^2$ (iii) $\sum \alpha^3$ (iv) $(\alpha + \beta)(\beta + \gamma)(\lambda + \alpha)$

OR

   b   Solve : $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$

17 a   Show that the equations $3x + 4y + 5z = a$, $4x + 5y + 6z = b$ and $5x + 6y + 7z = c$ don't have a solution unless $a + b = 2b$. Solve the equations when $a = b$, $c = 1$.

OR

   b   Test whether the following equations are consistent :
   $x + y + z = 8$, $x - y + 2z = 6$, $3x + 5y - 7z = 14$ if found consistent, find a solution.

18 a Find $\dfrac{dy}{dx}$, when $x = \dfrac{3at}{1+t^3}$, $y = \dfrac{3at^2}{1+t^3}$.

OR

  b If $y = \dfrac{2t}{1-t^2}$ and $\sin x = \dfrac{2t}{1+t^2}$, find $\dfrac{dy}{dx}$.

19 a If $y = x^2 e^x$, find $y_n - \frac{1}{2} n\,(n-1)y_2 - n\,(n-2)y_1 + \frac{1}{2}\,(n-1)(n-2)y$.

OR

  b If $y^{1/m} + y^{-1/m} = 2x$, find $(x^2 - 1)y_{n+2} + (2n+1)x\,y_{n+1} + (n^2 - m^2)y_n$.

20 a Evaluate $\displaystyle\int \dfrac{x^3 \sin^{-1} x}{\sqrt{1-x^2}}\,dx$.

OR

  b Evaluate $\displaystyle\int_0^\pi \left(\dfrac{x \tan x}{(\sec x + \tan x)} \right) dx$

Z-Z-Z   END