14MCU24

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(Sixth Semester)

Branch - **MATHEMATICS WITH COMPUTER APPLICATIONS**

LINEAR ALGEBRA

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10x2 = 20)

1 Define orthogonal matrix with example.

2 Find the rank of a matrix
1 2 3
2 4 6
3 6 9,

3 Prove that L(S) is a subspace of V.

4 Define annihilator of W.

5 Define an algebraic number.

6 State Remanider theorem.

7 Define an algebra over F.

8 Define matrix of T.

9 Define index of nil-potence of T.

10 Prove that if SeA(V) and if VSS* = 0, then VS = 0.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5x5= 25)

11 a Find inverse of
(3 3 4"
2 -3 4 .
lo -1 ij

OR

b Prove that any square matrix is expressible as a sum of a symmetric and skew - symmetric matrix.

12 a State and prove Schwarz's inequality.

OR

b Prove that if V is a vector space over F, then (i) a.O = 0 for a e F (ii) (-a)v = -(av) for veV (iii) if v * 0, then av = 0 => a = 0.

13 a Prove that if a, b in K are algebraic over F, then a ± b, ab and a/b are algebraic over F.

OR

b Prove that a polynomial of degree n over a field can have at mort n roots in any extension field.

Cont ..

14 a Prove that the element X e F is a characteristic root of T e A(V) if and only if for some v * 0, in V, then vT = *Xv.*

OR

b Let V be the vector space of polynomials of degree 3 (or) less over F. In V, define T by (a_0 + ot]X + a_2x^2 + a_3x^3)T = ao + aj(x+l) + $a_2(x+l)^2$ +$a_3(x+l)^3$. Compute the matrix of T in the basis 1, x, x^2, x^3.

15 a Prove that if (vT, vT) = (v,v) for all v e V, then T is unitary.

OR

b Prove that if N is normal and AN = NA, then AN* = N*A.

SECTION - C (30 Marks!

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Find the characteristic roots and characteristic vectors of $\begin{matrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{matrix}$

17 State and prove Gram - Schmidt orthogonalization process.

18 Prove that the element a e K is algebraic over F if and only if F(a) is a finite extension of F.

19 Prove that if V is finite dimensional over F, then for S, T e A(V), (i) r(ST) < r(T) (ii) r(TS) < r(T); (iii) r(ST) = r(TS) = r(T) for S regular in A(V).

20 Prove that if V is n-dimensional over F and if TeA(V) has all its characteristic roots in F, then T satisfies a polynomial of degree n over F.

Z-Z-Z END