PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(Third Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER SERIES

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. The equation z=(x+a) (y+b) gives the partial differential equation _____.
   (i) z=pq (ii) z=(p+a)(q+b)
   (iii) z=(x+p)(y+q) (iv) z=p+q

2. $(x-a)^2+(y-b)^2+z^2=1$ is a _____ integral of $z^2(1+p^2+q^2)=1$.
   (i) singular (ii) complete
   (iii) general (iv) circular

3. If u=f(x+iy)+g(x-iy), then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$ ______.
   (i) $f''(x+iy)+g''(x-iy)$ (ii) $f''(x+iy)-ig''(x-iy)$
   (iii) 0 (iv) $f(x+iy)-g(x-iy)$

4. _______ is called the two dimensional biharmonic equation.
   (i) $\nabla_1^2\phi=0$ (ii) $\nabla^2\phi=0$
   (iii) $\nabla_1\phi^2=0$ (iv) $\nabla_1^4\phi=0$

5. In $(-\pi,\pi)$, the Fourier coefficient a_0=______.
   (i) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$ (ii) $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$
   (iii) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$ (iv) $\int_{-\pi}^{\pi} f(x)dx$

6. Which of the following is an even function?
   (i) x sinx (ii) x+sinx
   (iii) x cosx (iv) x^2sinx

7. Fourier integral is mainly used to ______.
   (i) find the length of a curve (ii) find the area between two curves
   (iii) solve differential equations (iv) solve difference equations

8. If f(x) is an even function, then the value of B(w) in the Fourier integral is __.
   (i) 0 (ii) $\frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\sin wv\,dv$
   (iii) $\frac{2}{\pi}\int_{-\infty}^{\infty} f(v)\sin wv\,dv$ (iv) 1

9. $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ is called the _____ equation.
   (i) two-dimensional heat (ii) one-dimensional heat

10 The boundary value problem is called a Dirichlet problem if _____.
(i) u is prescribed on C (ii) u is prescribed on R
(iii) u=0 (iv) u is a constant

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that the general solution of the linear partial differential equation $Pp+Qq=R$ is $F(n,\nu)=0$ where F is an arbitrary function and $u(x,y,z)=c_1$ and $\nu(x,y,z)=c_2$ form a solution of the equations $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$.

OR

b If u is a function of x,y and z which satisfy the partial differential equation $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$, show that u contains x,y and z only in combinations of $x+y+z$ and $x^2+y^2+z^2$.

12 a Find the solution of the equation $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=x-y$.

OR

b If $u=f(x+iy)+g(x-iy)$, where the functions f and g are arbitrary, show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.

13 a Find the Fourier series of the function $f(x)=\begin{cases}0 & \text{if} \quad -2<x<-1\\ k & \text{if} \quad -1<x<1\\ 0 & \text{if} \quad 1<x<2\end{cases}$

OR

b Find the Fourier cosine series of the function $f(x)=\pi-x \ (0<x<\pi)$.

14 a Let f(x) be continuous and absolutely integrable on the x-axis, $f^1(x)$ be piecewise continuous on each finite interval and $f(x)\to 0$ as $x\to\infty$. Then prove:

$$\text{(i)}\ \mathfrak{F}_c\{f^1(x)\}=w\mathfrak{F}_s\{f(x)\}-\sqrt{\frac{2}{\pi}}f(0) \qquad \text{(ii)}\ \mathfrak{F}_s\{f'(x)\}=-w\mathfrak{F}_c\{f(x)\}.$$

OR

b State and prove the convolution theorem.

15 a Find the temperature u(x,t) in a laterally insulated copper bar 80cm long if the initial temperature is $100\sin\left(\frac{3\pi x}{80}\right)^{\circ}C$ and the ends are kept at $0^{\circ}C$. How long will it take for the maximum temperature in the bar to drop to $50^{\circ}C$? Physical data for copper: density 8.92 gm/cm^3, specific heat 0.092 $cal/(gm^{\circ}C)$, thermal conductivity 0.95 $cal/(cm\ sec^{\circ}C)$.

OR

b Suppose the initial temperature is $u(x,0)=f(x)$ and the boundary condition is $u(0,t)=0$. Find the temperature in the laterally insulated bar using the Fourier sine transform.

SECTION -C (40 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a (i) Find the general integral of the equation: $(xp-yp)=y^2-x^2$
(ii) Find the integral surface of the linear partial differential equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight line $x+y=0$, $z=1$.

OR

b Find the surface which is orthogonal to the one-parameter system $z=cxy(x^2+y^2)$ and which passes through the hyperbola $x^2-y^2=a^2$, $z=0$.

17 a Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

OR

b Find a particular integral of the equation:
(i) $(D^2-D^1)z=2y-x^2$ (ii) $(D^2-D^1)z=e^{x+y}$

18 a Find the Fourier cosine series as well as Fourier sine series of the function $f(x)=x$, $(0<x<\pi)$.

OR

b Find the Fourier series of the function $f(x)=x+\pi$ if $-\pi<x<\pi$ and $f(x+2\pi)=f(x)$.

19 a Find the Fourier sine integral of $f(x)=e^{-kx}$ and hence show that
$\int_0^\infty \frac{w \sin wx}{k^2+w^2} dw = \frac{\pi}{2} e^{-kx}$

OR

b Find the Fourier transform of (i) $f(x) \begin{matrix} =k & \text{if} & 0<x<a \\ =0 & \text{otherwise} & \end{matrix}$ (ii) $f(x)=xe^{-x^2}$

20 a Solve the one-dimensional heat equation by the method of separation of variables.

OR

b Using the Fourier transform, find the temperature in the infinite bar if the initial temperature is $f(x) = \begin{cases} U_0 = \text{constant} & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases}$.

Z-Z-Z END