18MCU02

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ORDINARY DIFFERENTIAL EQUATIONS AND LAPLACE TRASNFORMS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 Find the solution of the initial value problem $\frac{dy}{dx} = 2x + 1; y(0) = 3$.

(i) $y(x)=x^2+x$ (ii) $y(x)=x^2+x+3$ (iii) $y(x) = \frac{2x^2}{3} + x$ (iv) $y(x)=(2x+1)^2+3$

2 Which of the following is not an exact differential equation?
(i) $y^3dx+3xy^2dy=0$ (ii) $ydx+3xdy=0$
(iii) $(2x+3y)dx+13x+2ydy=0$ (iv) $(e^x\sin y+\tan y)dx+(e^x\cos y+x\sec^2 y)=0$

3 Which one of the following statements are true?
(i) The Wronskian of two linearly independent function is everywhere zero
(ii) The Wronskian of two linearly independent function is everywhere non zero
(iii) The Wronskian of two linearly dependent function is everywhere non zero
(iv) The Wronskian of two linearly independent function is identically zero

4 Find the general solution of $y^{11}+2y^1=0$.
(i) $y(x) = ce^x + c_2e^{-2x}$ (ii) $c_1e^{2x} + c_2e^{-2x}$ (iii) $c_1 + c_2e^{-2x}$ (iv) $c_1 + c_2e^{2x}$

5 The roots of the characteristic equation of a certain differential equation are $0,0,0,\frac{1}{3},\frac{1}{3}$. What is its general solution?

(i) $y(x) = c_1 + c_2x + c_3x^2 + c_4e^{x/3} + c_5xe^{x/3}$ (ii) $y(x) = c_1 + c_2x + c_3x^2 + c_4e^{x/3} + c_5e^{x/3}$

(iii) $y(x) = c_1e^{x/3} + c_2xe^{x/3}$ (iv) $y(x) = c_1e^{x/3} + c_2e^{x/3}$

6 A differential equation of order 3 can be written as a system of _____ differential equation of order one.
(i) 2 (ii) 3 (iii) 4 (iv) 9

7 Find $\Gamma\left(\frac{5}{2}\right)$

(i) $\sqrt{\pi}$ (ii) $5\sqrt{\pi}$ (iii) $\frac{4}{3}\sqrt{\pi}$ (iv) $\frac{3}{4}\sqrt{\pi}$

8 $L^{-1}\left[\frac{1}{(s-a)^{n+1}}\right]$ is

(i) $\frac{t^n e^{at}}{n!}$ (ii) $\frac{t^n e^{-at}}{n!}$ (iii) $t^n e^{-at}$ (iv) $t^n e^{at}$

9 The convolution of cos t and sin t is

(i) $t \sin t$ (ii) $\frac{t}{2}\sin t$ (iii) $t \cos t$ (iv) $\frac{t}{2}\cos t$

10 What is $L[(1_a lt)]$ if $a > 0$?

(i) $\frac{e^{-as}}{s}$ (ii) $\frac{e^{-as}}{a}$ (iii) $\frac{e^{as}}{s}$ (iv) $\frac{e^{as}}{a}$

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 Cont...

b Suppose that at time t = 0, 10 thousand people in a city with population M=100 thousand have heard a certain rumor. After 1 week the number p(t) of those who have heard it has increased to p(1)=20 thousand. Assuming that p(t) satisfies a logistic equation, when will 80% of the city's population have heard the rumor?

12 a State and prove the principle of super position for homogenous equations.

OR

b Find the general solution the initial value problem $y^{11}+4y=12x$, $y(0)=5$, $y^{1}(0)=7$.

13 a Find a particular solution of $y^{11}-4y=2e^{2x}$.

OR

b Solve the two-dimensional system $x^{1}=-2y$, $y^{1}=\frac{1}{2}x$.

14 a Find L[t sin kt].

OR

b Find $L^{-1}\left\{\dfrac{s^2+1}{s^3+2s^2+8s}\right\}$.

15 a Suppose that f(t) and g(t) are piecewise continuous for $t \geq 0$ and that $|f(t)|$ and $|g(t)|$ are bounded by Me^{ct} as $t \to +\infty$. Then prove that the laplace transform of the convolution f(t)*g(t) exists for s>c; More over L{f(t)*g(t)}=L{f(t)}*L{g(t)} and L^{-1}\{F(s).G(s)\}=f(t)*g(t).

OR

b Find L{g(t)} if $g(t)=\begin{cases} 0 & \text{if} \quad t<3 \\ t^2 & \text{if} \quad t \geq 3 \end{cases}$

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a (i) Solve the differential equation $2xy\dfrac{dy}{dx}=4x^2+3y^2$.

(ii) Suppose that the river is 1 mile wide and that its midstream velocity is $v_0 = g\,\text{mi/h}$. gf the swimmer's velocity is $v_s = 3\,\text{mi/h}$, find the swimmer's trajectory and the distance he has to drift to cross the river.

OR

b Derive the equation of the plane's trajectory.

17 a (i) Let y_1 and y_2 be two linearly independent solutions of the homogenous equation $y^{11}+p(x)y^{1}+q(x)y=0$ with p and q continuous on the open interval I. If Y is any solution of this equation on I, then prove that there exist numbers c_1 and c_2 such that $Y(x)=c_1y_1(x)+c_2y_2(x)$.

OR

b Solve the initial value problem $y^{11}+2y^{1}+y=0$; $y(0)=5$; $y^{1}(0)=-3$.

18 a Solve the initial value problem $y^{(3)}+3y^{11}-10y^{1}=0$; $y(0)=7$; $y^{1}(0)=0$, $y^{11}(0)=70$.

OR

b Consider an RLC circuit with R=50 ohms (Ω), L=0.1 Henry (H) and C=$5\text{x}10^{-4}$ farad(F). At time t=0, when both I(O) and Q(O) are zero, the circuit is connected to a 110-v, 60-Hz alternating current generator. Find the current in the circuit and the time lag of the steady periodic current behind the voltage.

19 a Solve the initial value problem $x^{11}+4x=\sin 3t$; $x(0)=x^{1}(0)=0$ using Laplace transform.

OR

b Solve the initial value problem $y^{11}+4y^{1}+4y=t^2$; $y(0)=y^{1}(0)=0$ using Laplace transform.

20 a (i) Find $L\left\{\dfrac{\sinh t}{t}\right\}$. (ii) Find $L^{-1}\left\{\tan^{-1}\left(\dfrac{1}{s}\right)\right\}$.

OR