**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)
**BSc DEGREE EXAMINATION DECEMBER 2019**
(Fifth Semester)
Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**
**ABSTRACT ALGEBRA**

Time : Three Hours Maximum : 75 Marks

**SECTION-A (20 Marks)**
Answer **ALL** questions
**ALL** questions carry **EQUAL** marks (10 x 2 = 20)

1 Define a subgroup of a group.
2 Define the right coeset of a subgroup at an element in a group.
3 Define isomorphism on groups.
4 State Cayley's theorem.
5 Define an integral domain.
6 Define a homomorphism on rings.
7 Define a Euclidean ring.
8 Define relatively primes.
9 Define the degree of a polynomial.
10 Define a primitive polynomial.

**SECTION - B (25 Marks)**
Answer **ALL** Questions
**ALL** Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that, a nonempty subset H of a group G is a subgroup if and only if (i) a, b $\in$ H implies that ab $\in$ H (ii) a $\varepsilon$ H implies that $a^{-1} \in$ H

OR

b List the three left coeset and three right cosets of H = { e, $\phi$} in $S_3$.

12 a Define $\phi$: G→ $\overline{G}$ by $\phi(x) = \log_{10}x$, where G is the group of positive real numbers under multiplication and $\overline{G}$ is the group of all real numbers under addition. Show that $\phi$ is a homomorphism, one-one and onto.

OR

b Find the cycles of the permutations $\theta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 8 & 1 & 6 & 4 & 7 & 5 & 9 \end{pmatrix}$.

13 a If $\phi$ is a homomorphism of R into R' with kernel I($\phi$), then prove that (i) I($\phi$) is a subgroup of R under addition. Ii) If a $\varepsilon$ I($\phi$), and r $\varepsilon$ R, then both ar and ra are in I($\phi$),

OR

b Define associative ring. Give an example

14 a Let R be the ring of integers and let U =($n_0$), where $n_0$ is a fixed integer, be an ideal of R. For what values of $n_0$, U is a maximal ideal? Justify your answer.

OR

b Let R be an integral domain with unit element and suppose that for a, b $\varepsilon$ R both b/a and a/b are true. Then, Prove that a = ub, where u is a unit in R.

15 a State and prove the division algorithm.

OR

b If f(x) and g(x) are primitive polynomials, then prove that f(x)g(x) is a primitive polynomial.

**Conti…**

**SECTION - C (30 Marks)**

Answer any **THREE** Questions

**ALL** Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 If H and K are finite subgroups of G of orders O(H) and O(K) respectively, prove that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$.

17 State and prove Sylow's theorem for abelian groups.

18 Prove that a finite integral domain is a field.

19 State and prove unique factorization theorem.

20 R is a unique factorization domain, prove that R[x] is a unique factorization domain.

Z-Z-Z END