

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(Third Semester)

Branch - STATISTICS

STATISTICAL INFERENCE -I

Time: Three Hours

Maximum: 75 Marks

Answer ALL questions
ALL questions carry EQUAL marks (5 x 15 = 75)

- 1 a Define sufficient statistics and mention its significance. Explain with examples the procedures of verifying whether given statistics possess these properties.
- b Prove that consistency is invariant under continuous transformations.
- OR
- c State and prove the Factorization theorem.
- d Describe the following : (i) minimal sufficiency (ii) sufficient statistics
- 2 a State and prove Kiefer - Chapman - Robbins inequality.
- b Write a detailed note on Bhattacharya bounds.
- OR
- c Describe the following : (i) Efficiency (ii) UMVUE
- d State and prove Lehman - Scheffe's theorem.
- 3 a Write a short note on method of moments.
- b Prove that the minimum chi-square estimate of θ is that value $\hat{\theta}(\theta \in \Theta)$ satisfying $C(\hat{\theta}) < C(\theta)$, for all $\theta \in \Theta$.
- OR
- c Let X_1, X_2, \dots, X_n be a sample from $U(0, \theta)$, $\theta > 0$. The likelihood function is

$$L(\theta; x_1, x_2, \dots, x_n) = \begin{cases} \theta^{-n} & \text{if } 0 < \min(x_1, \dots, x_n) < \max(x_1, \dots, x_n) \leq \theta \\ 0 & \text{otherwise} \end{cases}$$
 Find MLE of θ .
- d Write down the small and large sample properties of MLE.
- 4 a Explain the location and scale invariant estimators with examples.
- OR
- b Obtain the posterior distribution of θ in the Bernoulli (θ) distribution employing the Jeffreys non informative prior to θ . Also, find the Bayes estimator for θ under squared error loss function.
- c Describe Pitman estimator.
- 5 a Discuss the large sample confidence interval.
- b Obtain the $100(1-\alpha)\%$ shortest length confidence interval for θ for the case of uniform $(0, \theta)$ distribution.
- OR
- c Define the following :
(i) lower confidence limit (ii) confidence bounds
- d Explain briefly uniformly most accurate confidence bounds.