

PSG COLLEGE OF ARTS & SCIENCE  
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018  
(First. Semester)

Branch - STATISTICS

PROBABILITY THEORY

Time: Three Hours

Maximum; 75 Marks

Answer ALL questions  
ALL questions carry EQUAL marks (5 x 15 = 75)

- 1 a i) If  $X$  and  $Y$  are simple random variables then prove that  $E(X \pm Y) = E(X) \pm E(Y)$ .  
 ii) If two non-decreasing sequences of non-negative simple function  $\{X_n\}$  and  $\{Y_n\}$  have the same limit  $X$ , then  $\lim E(X_n) = \lim E(Y_n) = E(X)$ .  
 OR  
 b State and prove :  
 i) CR Inequality and (ii) Markov's inequality
- 2 a i) Define characteristic function.  
 ii) State and prove any three properties of characteristic function.  
 OR  
 b Prove the second limit theorem.
- 3 a i) Define Class of Independent Events,  
 ii) If  $A$  and  $B$  are independent events, then  $(A \text{ and } B)$ ,  $(A \text{ and } B^c)$  are independent.  
 OR  
 b i) Prove the Kolmogorov 0-1- law.  
 ii) If  $X_n$ 's are independent and  $X_n \rightarrow 0$  (a. s), then  $P[\sum_{j=1}^n X_j > c] < \infty$ ,
- 4 a If  $X_n \xrightarrow{P} X$ ,  $Y_n \xrightarrow{P} Y$ , then prove that  $X_n \xrightarrow{P} aX$  ( $a$  is real number).  $X_n + Y_n \xrightarrow{P} X + Y$ ,  $X_n Y_n \xrightarrow{P} XY$  and  
 when  $P(Y_n = 0) = 0 \forall n$  and  $P(Y = 0) = 0$ .  
 OR  
 b i) Define convergence in  $r$  mean,  
 ii) If  $X_n \xrightarrow{P} X$ , then  $E|X_n|^r$
- 5 a State and prove the Kolmogorov strong law of large numbers for i. i. d case.  
 OR  
 b i) State, and prove Lindeberg - Levy theorem  
 ii) State the Liapounov's theorem.