Exam Date & Time: 26-Sep-2020 (10:00 AM - 01:30 PM)

PSG COLLEGE OF ARTS AND SCIENCE

Note: Writing 3hrs: Checking & Inserting Image : 30mins

BSc DEGREE EXAMINATION MAY 2020
(Sixth Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS

LINEAR ALGEBRA [14MCU24]

Marks: 75 **Duration: 210 mins.**

SECTION A

Answer all the questions.

1) If A is non singular and B commutes with A, then show that B commutes with A^{-1}. (2)

2) Define characteristic polynomial and characteristic equation of a square matrix A. (2)

3) Define a vector space homomorphism. (2)

4) Prove that $w^{\perp}$ is a subspace of v. (2)

5) Define an algebraic number. (2)

6) Define a root of multiplicity m. (2)

7) Define algebra. (2)

8) Define the matrix of a linear transformation T. (2)

9) Define similar transformations. (2)

10) Define the Hermitian adjoint of a linear transformation T. (2)

SECTION B

Answer all the questions.

11)

a) Show that $\frac{1}{2}\begin{bmatrix}1+i & -1+i \\ 1+i & 1-i\end{bmatrix}$ is an unitary matrix. (5)

[OR]

b) Find the characteristic roots of the matrix $A=\begin{pmatrix}1 & 0 & 2\\ 0 & 1 & 2\\ 1 & 2 & 0\end{pmatrix}$. (5)

12)

a) (i) Define linearly independent set.
(ii) If $v_1, v_2, \dots v_n$ are in V then prove that either they are linearly independent or some v_k is a linear combination of the preceding ones $v_1, v_2, \dots v_{k-1}$. (5)

[OR]

b) If u, v∈v then prove that $|(u, v)|\leq\|u\|\,\|v\|$. (5)

13)

a) If a,b in k are algebraic over F then prove that a±b, ab and a/b (if b≠0) are all algebraic over F. (5)

[OR]

b) State and prove the remainder theorem. (5)

14)

a) If V is finite dimensional over F, prove that T∈A(V) is invertible if and only if the constant term of the minimal polynomial for T is not zero. (5)

[OR]

b) If V is finite dimensional over F, prove that T∈A(V) is regular if and only if T maps V onto V. (5)

15)

a) If V is n-dimensional over F, prove that T∈A(V) has all its characteristic roots in F, prove that T satisfies a polynomial of degree n over F. (5)

[OR]

b) If T∈A(V) is such that (vT, v)=0 for all v∈V, prove that T=0. (5)

SECTION C

Answer 3 out of 5 questions.

16) State and prove the Cayley-Hamilton theorem. (10)

17) (10)

If V and W are of dimensions m and n, respectively over F, prove that Hom(V,W) is of dimension mn over F.

18) If L is a finite extension of K and if K is a finite extension of F prove that L is a finite extension of F. (10)

19) If V is finite dimensional over F then S, T∈A(V) prove

(i) r(ST)≤r(T)

(ii) r(TS)≤r(T) and (10)

(iii) r(ST)=r(TS)=r(T) for S regular in A(V).

20) If T∈A(V) has all its characteristic roots in f, prove that there is a basis of V in which the matrix of T is triangular. (10)

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