PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(Fourth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

SEQUENCE, SERIES AND TRIGONOMETRY

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 The series $\sum_{n=1}^{\infty}\frac{n^2}{5n^2+4}$ is ________.
(i) convergent (ii) divergent
(iii) monotonic (iv) decreasing

2 For the series with positive terms $\sum a_n$ and $\sum b_n$, $\sum b_n$ is divergent implies $\sum a_n$ is also divergent if______.
(i) $a_n \le b_n$ (ii) $a_n \ge b_n$
(iii) $a_n < b_n$ (iv) $a_n = b_n$

3 A series $\sum a_n$ is called ______ if it is convergent but not absolutely convergent.
(i) absolutely convergent (ii) absolutely divergent
(iii) conditionally convergent (iv) conditionally divergent

4 If $\lim_{n\to\infty}\sqrt[n]{an} = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent. This test is called ______.
(i) Root test (ii) Ratio test
(iii) Comparison test (iv) Integral test

5 The Interval of convergence for geometric series is_______.
(i) (0,1) (ii) [0,1]
(iii) (-1,1) (iv) [-1,1]

6 The power series of $\frac{1}{(1-x)^2}$ is ______.
(i) $1+x+x^2+....$ (ii) $1+2x+3x^2+....$
(iii) $1-x-x^2-....$ (iv) $1+2x-3x^2+....$

7 The series expansion of $\sin\theta$ is___________.
(i) $1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-....$ (ii) $1+\frac{\theta^2}{2!}+\frac{\theta^4}{4!}+....$
(iii) $\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-....$ (iv) $\theta+\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-....$

8 The value of $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} =$ ________.

(i) 0
(ii) ∞
(iii) 0
(iv) indeterminate form

9 The expression for sin hx = ________.

(i) $\frac{e^{x} + e^{-x}}{2i}$
(ii) $\frac{e^{ix} + e^{-ix}}{2i}$
(iii) $\frac{e^{ix} - e^{-ix}}{2i}$
(iv) $\frac{e^{x} - e^{-x}}{2i}$

10 The value of sin (ix) = ________.

(i) i sin x
(ii) i sin hx
(iii) sin hx
(iv) None of these

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Show that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is convergent and find its sum.

OR

b Show that the sequence $a_n = \frac{n}{n^2+1}$ is decreasing.

12 a Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

OR

b Test the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ for absolute convergence.

13 a For what value of X is the series $\sum_{n=1}^{\infty} n!x^n$ convergent?

OR

b Find the power series representation for $f(x) = \tan^{-1}(x)$.

14 a Expand $\sin^6 \theta$ in series of cosines of multiples of θ.

OR

b Find $\lim_{\theta \to 0} \frac{n\sin\theta - \sin n\theta}{\theta(\cos\theta - \sin n\theta)}$.

15 a Prove that tan hzx= $\frac{2\tanh x}{1+\tanh^2 x}$

OR

b Show that $\log(i) = \frac{4n+1}{4m+1}$, where m and n are integers.

Cont...

SECTION -C (40 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Prove that every bounded monotomic sequence is convergent.

OR

b Approximate the sum of the series $\sum 1/n^3$ by using the sum of the first 10 terms. Estimate the error inrolved in this approximation.

17 a Test the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{n^3+1}$ for convergence or divergence.

OR

b State and prove the Ration test.

18 a Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$.

OR

b For what value of x does the series $\sum_{n=1}^{\infty}\frac{(x-3)^n}{n}$ converge ?

19 a Separate into real and imaginary parts $\tan^{-1}(x+iy)$.

OR

b Expand $\sin^3\theta\cos^5\theta$ in a series of sines of multiples of θ.

20 a Prove that if $\log(\theta+i\phi)$= A+ib then $2e^{2A}=\log 2\phi-\cos(2\phi)$..

OR

b If $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$, find the sum to infinity of the series $1+\frac{1}{2}\cos 2\theta-\frac{1}{2.4}\cos 4\theta+\frac{1.3}{2.4.6}\cos 6\theta.....$

Z-Z-Z

END