18MCU29

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

COMPLEX ANALYSIS

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 When the limit of the function f(z) exists as z tends Z_0 then the limit has
(i) unique value (ii) distinct value
(iii) none (iv) Real Value

2 if f= u+iv be an analytic function in a region D then the v is ....
(i)conjugate harmonic (ii) partial order
(iii) real constant (iv) None

3 Statement I: $\int_a^b |f(t)|dt \leq \left|\int_a^b f(t)dt\right|$ where $f(t)$ is a continuous complex valued function defined on $[a,b]$. Statement II: $\int_{-C} f(z)dz = -\int_C f(z)dz$ Which of the above statements is not true?
(i) I alone (ii) II alone
(iii) Both I and II (iv) Neither I nor II

4 When $f(z)$ is analytic inside and on C and z_0 is any interior of C, then $f(z_0) =$
(i) $\frac{1}{\pi i}\int_C \frac{f(z)}{(z-z_0)}dz$ (ii) $\pi i \int_C \frac{f(z)}{(z-z_0)}dz$
(iii) $2\pi i \int_C \frac{f(z)}{(z-z_0)}dz$ (iv) $\frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_0)}dz$

5 The Taylor series expansion of f(z) about the point zero is called the ......Series
(i) Maclarin's (ii) Power
(iii) Constant (iv) None

6 The Maclaruin series is ____________
(i) $f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} z^n$ (ii) $f(z) = \frac{f^{(n)}(0)}{n!} z^n$
(iii) $\frac{f^{(n)}(0)}{n} z^n$ (iv) None

7 If f(z) is not analytic at a then a is called a ....
(i) Regular (ii) Singular point
(iii) Critical point (iv) Zero Point

8 Let a be an isolated singularity for f(z) then a is called a...... Singularity. If the principal part of f(z) at z=a has no terms.
(i) Poles (ii) essential
(iii) Removable (iv) None

9 Another name of a pole is ________
(i) point (ii) order (iii)Curve (iv)None

10 Find the residue of cot z at z= 0 *is*
(i) 0 (ii)1 (iii) π (iv) None

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a If $f(z) = \frac{z}{\bar{z}}$ then prove that $\lim_{z \to 0} f(z)$ does not exist.

OR

Cont...

b. If $f'(z) = 0$ everywhere in a domain D, then show that f(z) must be constant throughout D.

12 a. Find the value of the integral $I = \int_C \bar{z}\, dz$ when C is the right – hand half $z = 2e^{i\theta}\left(-\pi/2 \le \theta \le \pi/2\right)$ of the circle $|z| = 2$, from z - 2i to z = 2i.

OR

b. Evaluate the $\int_0^{\pi/6} e^{i2t}\, dt$.

13. a. If C is the positively oriented circle $|z| = 2$, then find $f(z) = \dfrac{z}{9-z^2}$.

OR

b. State and prove Liouville's theorem.

14 a. Evaluate $\int_C \dfrac{dz}{z(z-2)^4}$, where C is the positively oriented circle $|z - 2| = 1$.

OR

b. Find the residue at z = 0 of the function $\dfrac{z - \sin z}{z}$.

15 a. Determine the order m of each pole, and find the corresponding residue B $\dfrac{z^2+2}{z-1}$.

OR

b. Find the Cauchy principal value of the integral $\int_{-\infty}^{\infty} \dfrac{x \sin x\, dx}{x^2+2x+2}$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a. Derive Cauchy – Riemann equations.

OR

b. If a function f(z)= u(x, y) + iv(x, y) is analytic in a domain D, then prove that its component functions u and v are harmonic in D.

17 a. State and prove Cauchy – Goursat theorem.

OR

b. Use an antiderivative to evaluate the integral $\int_{C_1} z^{1/2}\, dz$, where the integrand is the branch $z^{1/2} = \sqrt{r}e^{i\theta/2}$ (r > 0, 0 < θ < 2π)

18 a. State and prove cauchy Integral formula.

OR

b. State and prove Taylor's theorem.

19 a. State and prove Cauchy's residue theorem.

OR

b. If a function f is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, then prove that

$$\int_C f(z)dz = 2\pi i \operatorname*{Res}_{z=0}\left[\frac{1}{z^2} f\left(\frac{1}{z}\right)\right]$$

20 a. If a function f is analytic and bounded in some deleted neighborhood $0 < |z - z_0| < \epsilon$ of a point z_0. If f is not analytic at z_0, then prove that it has a removable singularity there.

OR

b. Evaluate $\int_0^{\infty} \dfrac{x^2}{x^6+1}\, dx$.

Z-Z-Z END