18MCU31

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ADVANCED MATHEMATICAL STATISTICS

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 The random variable x has a two-point distribution if there exist two values x_1 *and* x_2 such that____________ $(0 < p < 1)$.
(i) $P(X = x_1) = p$
(ii) $P(X = x_2) = 1 - p$
(iii) $P(X = x_0) = 1$
(iv) i & ii

2 The random variables x and Y are said to be independent if for every pair (x, y) of real numbers, ____ is satisfied.
(i) $F(x, y) \neq F_1(x)F_2(y)$
(ii) $F(x, y) = F_1(x)F_2(y)$
(iii) $F(x, y) \leq F_1(x)F_2(y)$
(iv) $F(x, y) \geq F_1(x)F_2(y)$

3 _________ is the first moment of X in Polya distribution.
(i) np
(ii) $np(A + B)$
(iii) $np(A - B)$
(iv) $np(AB)$

4 The random variable X has the beta distribution with p=q=2 and density

$$f(x) = \begin{cases} 0 \quad for \ y \leq 0 \ and \ y \geq 1 \\ \dfrac{\Gamma(4)}{\Gamma(2)\Gamma 92)} y(1 - y) = 6y(1 - y) \ \ for \ \ 0 < y < 1 \end{cases}$$

and ____ is the probability that X is not greater than 0.2?
(i) 0.204
(ii) 0.240
(iii) 0.124
(iv) 0.104

5 If the variances of a sequence of _____ random variables are uniformly bounded.
(i) Dependent
(ii) Independent
(iii) Discrete
(iv) Probability

6 The sequence $\{X_n\}$ of random variables is called stochastically convergent to zero if for every $\in > 0$ the relation ____ is satisfied.
(i) $\lim_{n\to\infty} P(|X_n| < \in) = 0$
(ii) $\lim_{n\to\infty} P(|X_n| > \in) \leq 0$
(iii) $\lim_{n\to\infty} P(|X_n| > \in) \geq 0$
(iv) $\lim_{n\to\infty} P(|X_n| > \in) = 0$

7 _____theorem gives a sufficient condition for a sum of independence random variables to have a limiting normal distribution.
(i) Borel law
(ii) Kolmogorov law
(iii) Levy- Cramer
(iv) Lapunov

8 ____________ theorem gives the necessary and sufficient condition for the relation $\lim_{n\to\infty} F_n(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-z^2/2} dz$.
(i) Lapunov
(ii) Gnedenko
(iii) Lindeberg-Feller
(iv) Loeve

Cont...

9 ______ model only arrivals occur.

(i) Pure birth (ii) Pure death

(iii) (M/M/1): (GD/∞/∞) (iv) all

10 All the Ergodic probabilities equal ______

(i) ∞ (ii) 1

(iii) 0 (iv) non zero

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Derive $\int_{-\infty}^{\infty} f_1(x)F(y \mid x)dx = F_2(y)$

OR

b A real single-valued function $F(x,y)$ is a distribution function of a certain two-dimensional random variable iff $F(x,y)$ is nondecreasing and continuous at least from the left with respect to both arguments x *and* y satisfies equalities $F(-\infty,y) = F(x,-\infty) = 0$, $F(+\infty,+\infty) = 1$ and the inequality $F(x_2,y_2) - F(x_2,y_1) - F(x_1,y_2) + F(x_1,y_1) \geq 0$

12 a Find the characteristic function of Gamma distribution.

OR

b Find the characteristic function of Cauchy distribution.

13 a State and prove Levy-Cramer theorem.

OR

b Let $F_n(x)(n = 1,2,.......)$ be the distribution function of the random variable X_n. The sequence $\{X_n\}$ is stochastically convergent to zero iff the sequence $F_n(x)$ satisfies the relation $\lim_{n\to\infty} F_n(x) = \begin{cases} 0 & for\ x \leq 0 \\ 1 & for\ x > 0 \end{cases}$

14 a Let $X_k(k = 1,2,.......)$ be a sequence of independent random variables with the sdame distribution and with expected value $E(X_k) = m$ then prove that the sequence $\{Y_n\}$ where $Y_n = \dfrac{X_1 + X_2 + ... + X_n}{n}$ is stochastically convergent to m.

OR

14 b Let $\{X_k\}(k = 1,2,3,..........)$ be a sequence of independent, uniformly bounded variables, there exists a constant a>0 such that for every k $P(|X_k| \leq a) = 1$ and suppose that $D^2(X_k) \neq 0$ for every k, then a necessarily and sufficient condition for relation $\lim_{n\to\infty} F_n(z) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-z^2/2} dz$ to hold is $\lim_{n\to\infty} C_n^2 = \infty$

15 a Discuss the Yule-Furry process.

OR

b Explain Birth and Death Processes

SECTION -C (40 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Show that if (X,Y) is a random variable of the continuous type whose density function f(x,y) is everywhere continuous, the validity of $\frac{\partial^2 F(x,y)}{\partial x \partial y} = f(x,y) = F_1'(x)F_2'(y) = f_1(x)f_2(y)$ for arbitrary points (x,y) is a necessary and sufficient condition for the independence of the random variables X and Y.

OR

b Consider the independent random variables X and Y with the density of Gauss distribution. Find the density of the joint random variable Z (ie Z=X+Y).

17 a Explain Beta distribution.

OR

b Explain Uniform distribution

18 a State and prove de Moivre-Laplace theorem.

OR

b State and prove Bernoulli's Law of large numbers.

19 a State and prove Lindeberg-Levy theorem.

OR

b State and prove Lapunov theorem.

20 a Prove that the solutions $V_m(t)$ of the system $V_0'(t) = -\lambda_i V_0(t)$ and $V_m'(t) = -\lambda_{i+m} V_m(t) + \lambda_{i+m-1} V_{m-1}(t)$ with initial conditions $V_m(0) = \begin{cases} 1 & for \quad m = 0 \\ 0 & for \quad m \neq 0 \end{cases}$ satisfies the relation $\sum_{m=0}^{\infty} V_m(t) = 1$ iff $\sum_{m=0}^{\infty} \frac{1}{\lambda_{i+m}} = \infty$

OR

b A stochastic process $\{X_t, 0 \le t < \infty\}$ where X_t is the number of signals in the interval [0,t) satisfying conditions

(i) The process $\{X_t, 0 \le t < \infty\}$ is a process with independent increments.

(ii) The process $\{X_t, 0 \le t < \infty\}$ is a process with homogeneous Increments.

(iii) The following relations are satisfied $\lim_{t \to 0} \frac{W_1(t)}{t} = \lambda \ (\lambda > 0)$, $\lim_{t \to 0} \frac{1 - W_0(t) - W_1(t)}{t} = 0$ and the equality $P(X_0 = 0) = 1$ is a homogeneous Poisson process.

Z-Z-Z END