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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2022
(Fourth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ABSTRACT ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)
SECTION – A (10 x 1 = 10 Marks)

1 The two mappings σ and τ of S into Tare said to be equal if -------for every $s \in S$.
(i) $s\sigma = s\tau$ (ii) $s\sigma \geq s\tau$ (iii) $s\sigma \leq s\tau$ (iv) $s\sigma \neq s\tau$

2 A nonempty subset H of a group G is said to be a ---------of G if, under the product in itself forms a group.
(i) Right coset (ii) Congruence (iii) Modulo (iv) Subgroup

3 A subgroup N of G is said to be a ------------ of G if for every $g \in G$ and n $\in$N, $gng^{-1} \in N$.
(i) Normal subgroup (ii) Abelian group
(iii) Quotient group (iv) Subgroup

4 A homomorphism ϕ from G into $\bar{G}$ is said to be an isomorphism if ϕ is one-to-one
(i) Automorphisem (ii) Onto
(iii) Isomorphisem (iv) Homomorphism

5 Every permutation is the product of its----------
(i) m-cycle. (ii) Alternating group
(iii) Permutation (iv) Cycles

6 The product of two even permutations is an -------------
(i) Odd permutations (ii) Even permutation.
(iii) Positive permutations (iv) Negative permutations

7 If R is a commutative ring, then $a \neq 0 \in R$ is said to be a ------if there exists $a\ b \in R$, $b \neq 0$, such that ab = 0.
(i) Zero-divisor (ii) Commutative ring.
(iii) Division ring (iv) Integral domain

8 The homomorphism ϕ of R into R' is an ---------- if and only if $I(\phi) = (0)$.
(i) Transpositions. (ii) Permutation
(iii) Isomorphisem (iv) Homomorphism

9 Let R be a--------. Suppose that for $a, b, c \in R$, $a | bc$ but (a, b) = 1.Then a|c.
(i) Euclidean ring (ii) Commutative ring.
(iii) Division ring (iv) Integral domain

10 Let R be a ----------with unit element. An element $a \in R$ is a unit in R if there exists an element $b \in R$ such that ab = 1.
(i) Commutative ring . (ii) Associative ring
(iii) Division ring (iv) Euclidean ring

Cont...

SECTION - B (25 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a) If a and b are integers, not both 0, then *prove that* (a, b) exists; moreover, we can find integers m_0 and n_0 such that $(a, b) = m_0 a + n_0 b$.

(OR)

b) Prove that if G is a finite group and a ∈ G, then o(a)/ o(G).

12 a) Prove that if HK is subgroup of G if and only if HK=KH.

(OR)

b) Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

13 a) Prove that every permutation is the product of its cycles.

(OR)

b) Let G be a group and ϕ an automorphism in G. If $a \in G$ is if order $o(a) > 0$, then prove that $o(\phi(a)) = o(a)$.

14 a) If R is a ring, then for all a, b ∈ R, prove that

1. $a0 = 0a = 0$.
2. $a(-b) = (-a)b = -(ab)$.
3. $(-a)(-b) = ab$.

If in addition, R has a unit element 1, then

4. $(-1)a = -a$.
5. $(-1)(-1) = 1$.

(OR)

b) Define (i) zero-divisor (ii)integral domain (iii) field

15. a) Let R be a Euclidean ring. Then prove that every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R.

(OR)

b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.

SECTION -C (40 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a) Prove that Any positive integer $a > 1$ can be factored in a unique way as $a = p_1^{\alpha_1}, p_2^{\alpha_2} \dots p_t^{\alpha_t}$ where $p_1 > p_2 > \cdots > p_t$ are prime numbers and where each $\alpha_i > 0$.

(OR)

b) State and prove Lagranges theorem.

`17 a) State and prove fundamental theorem of homomorphism.

(OR)

b) State and prove Cauchy's theorem for Abelian groups.

18 a) State and prove Cayley's Theorem.

(OR)

b) Prove that if G is a group, then $\mathcal{A}(G)$, the set of automorphisms of G, is also a group

19 a) If p is a prime number then *prove that* J_p, the ring of integers mod p, is a field.

(OR)

b) Prove that finite integral domain is a field.

20 a) Prove that J[i] is a Euclidean ring.

(OR)

b) Prove that (i). If p is a prime number of the form 4n + 1, then $p = a^2 + b^2$ for some integers a, b. (ii).If p is a prime number of the form 4n + 1, then we can solve the congruence $x^2 = -1 \; mod \; p$.

Z-Z-Z END