8 a Find the differential coefficient of $\cot x$.

OR

b If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, find $\dfrac{dy}{dx}$.

9 a If $xy = ae^x + be^{-x}$, prove that $x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - xy = 0$.

OR

b Verify Euler's theorem when $u = x^3 + y^3 + z^3 + 3xyz$.

10 a Evaluate $\displaystyle\int \dfrac{x^3}{\sqrt{1 - x^8}}\,dx$.

OR

b Evaluate $\displaystyle\int x\sin 2x\,dx$.

SECTION -C (30 Marks)
Answer ALL questions
ALL questions carry EQUAL Marks   (5 x 6 = 30)

11 a Diminish the roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1 and hence solve the equation.

OR

b Solve the equation $2x^5 - 15x^4 + 37x^3 - 37x^2 + 15x - 2 = 0$.

12 a Calculate the transpose, adjoint and the inverse matrix of $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$.

OR

b Show that the equations $x + 2y = 3$, $y - z = 2$, $x + y + z = 1$ are consistent and solve them.

13 a Differentiate $\tan^{-1}\left(\dfrac{\cos x}{1 + \sin x}\right)$.

OR

b Differentiate (i) $y = (\sin x)^x$ and (ii) $y = a^x$.

14 a Find the nth differential coefficient of $\cos^5\theta \sin^7\theta$.

OR

b If $z = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that
$$\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 = \left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2.$$

15 a Evaluate $\displaystyle\int \dfrac{3x + 1}{(x - 1)^2(x + 3)}\,dx$.

OR

b Prove that $\displaystyle\int_0^{\pi/2} \dfrac{(\sin x)^{3/2}}{(\sin x)^{3/2} + (\cos x)^{3/2}}\,dx = \dfrac{\pi}{4}$.

Z-Z-Z   END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSC DEGREE EXAMINATION DECEMBER 2022

(Second Semester)

Branch – STATISTICS

MATHEMATICS - II

Time: Three Hours   Maximum: 50 Marks

SECTION-A (5 Marks)
Answer ALL questions

ALL questions carry EQUAL marks   (5 x 1 = 5)

1   The sum of the eigen values of the matrix $A = \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$ is

     (i) 1   (ii) 0   (iii) 8   (iv) 5

2   Which of the following partial differential equations is obtained from $z = ax + by + a$ by eliminating a, b ?

     (i) $z = px + qy$ (ii) $z = qx + py$

     (iii) $z = px + qy + q$ (iv) $z = px + qy + p$

3   When $f(x) = \frac{1}{2}(\pi - x)$ is expressed as a Fourier series in the interval $(0, 2\pi)$, then the value of a_0 is

     (i) 0   (ii) -1   (iii) 4   (iv) 1

4   The Laplace value of the function $t^2 + 2t + 3$ is

     (i) $\frac{2}{s^3} + \frac{2}{s^2}$ (ii) $\frac{2}{s^3} + \frac{2}{s^2} + \frac{3}{s}$

     (iii) $\frac{2}{s^3} + \frac{3}{s} + 2$ (iv) $\frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}$

5   Which of the following methods is known as direct method to solve simultaneous linear equations ?

     (i) Gauss Jacobi   (ii) Gauss Jordan

     (iii) Gauss elimination   (iv) none of these

SECTION - B (15 Marks)
Answer ALL Questions

ALL Questions Carry EQUAL Marks   (5 x 3 = 15)

6   a.   Find the eigen values of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

            OR

  b   Prove that if the matrix B is similar to the matrix A, then A and B have the same characteristic equation.

7   a   Eliminate the arbitrary functions f and φ from the relation $z = f(x + ay) + \varphi(x - ay)$.

            OR

  b   Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$

8   a   Express $f(x) = x$, $-\pi < x < \pi$ as a Fourier series with period 2π.

            OR

  b   Find a sine series for $f(x) = c$ in the range 0 to π.

9   a   Find $L^{-1} \left[\frac{1}{s(s+1)(s+2)} \right]$.

            OR