22PHU103

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(First Semester)

Branch – **PHYSICS**

MATHEMATICS - I

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The locus of the center of curvature for a curve is called the ----- of the curve.
   (i) Radius (ii) Evolute
   (iii) Elocute (iv) Coordinates

2. $\int_a^b x^3 dx =$
   (i) $\frac{a^4}{4} - \frac{b^4}{4}$ (ii) $\frac{b^4}{4} - \frac{a^4}{4}$
   (iii) $b^4 - a^4$ (iv) $a^4 - b^4$

3. $\int_0^2 \int_0^2 dxdy =$
   (i) ¼ (ii) 4
   (iii) 1/8 (iv) 8

4. The vector V is said to be solenoidal then ---
   (i) $\nabla \times V = 0$ (ii) $\nabla \times \nabla V = 0$
   (iii) $\nabla . V = 0$ (iv) $\nabla . \nabla V = 0$

5. Green's theorem states that
   (i) $\int_C Ndx + Mdy = \int_R (N_x - M_y)dx\,dy$
   (ii) $\iint_R Mdx + Ndy = \int_C (N_x - M_y)dxdy$
   (iii) $\int_C Mdx + Ndy = \iint_R (N_x - M_y)dx\,dy$
   (iv) $\int_C (M + N)dx = \iint_R (N_x M_y)dx\,dy$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at (1,1)?
   OR
   b. Find the coordinates of the center of curvature of the curve $xy = 2$ at (2,1).

7. a. Evaluate $\int x \sin 2x\, dx$.
   OR
   b. Calculate $\int_0^{\frac{\pi}{2}} sin^6 x cos^5 x\, dx$.

8. a. Compute $\int_0^a \int_0^b (x^2 + y^2)dx\,dy$.
   OR
   b. Compute $\int_0^3 \int_1^{5-x} x^2 y\, dx\, dy$.

Cont...

9. a. Find the directional derivative of the function $x^2 + y^2 + z^2$ at (3,6,9) in the direction whose direction cosines are $(\frac{1}{2}, \frac{2}{3}, \frac{2}{3})$.

OR

b. Show that the vector $A = x^2z^2i + xyz^2j - xz^3k$ is solenoidal.

10.a. Find the value of the integral $\int A.dr$, where $A = yzi + zxj - xyk$ for the curve whose parametric equations are $x = t, y = t^2, z = t^3$ from O(0,0,0) to Q(2,4,8).

OR

b. Show that $\iint r.n\, dS = 4\pi a^3$, if S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11.a. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + sin\theta)$ and $y = a(1 - cos\theta)$ is $4a\cos\frac{\theta}{2}$.

OR

b. Show that in the parabola $y^2 = 4ax$ at the point t, $\rho = -2a(1 + t^2)^{3/2}$, $X = 2a + 3at^3, Y = -2at^3$. Deduce the equation of the evolute.

12.a. Evaluate $\int x^3\, Cos\, 2x\, dx$.

OR

b. Evaluate $\int_0^{\frac{\pi}{2}} x(1 - x^2)^{\frac{1}{2}}\, dx$.

13. a. Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dx\, dy$ and evaluate it.

OR

b. Find the area of the cardioid $r = a(1 + \cos\theta)$.

14. a. Find $\nabla.\bar{F}$ and $\nabla \times \bar{F}$ at the point $(1, -1, 1)$ if $\bar{F} = xz^3\bar{\imath} - 2x^2yz\bar{\jmath} + 2yz^4\bar{k}$.

OR

b. If $\bar{F} = x^2y\bar{\imath} + y^2z\bar{\jmath} + z^2x\bar{k}$, then find curl curl $\bar{F}$.

15. a. State and prove Gauss' Divergence theorem.

OR

b. Evaluate by using Green's theorem $\int_C (xy + x^2)dx + (x^2 + y^2)dy$, where C is the square formed by the lines $x = -1, x = 1, y = -1, y = 1$ in the XOY plane.

Z-Z-Z END