

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Fifth Semester)

Branch – PHYSICS

MATHEMATICAL PHYSICS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 1 = 10)

- The divergence of a vector field is always a
(i) scalar (ii) vector (iii) numeric (iv) neither a scalar nor a vector
- A tensor of rank 2 in n-dimensional space has components.....
(i) n (ii) 2n (iii) n^2 (iv) 2^n
- In cylindrical co-ordinate system, the values of scale function h_1 , h_2 and h_3 are.....
(i) $h_1 = 1, h_2 = r, h_3 = 1$ (ii) $h_1 = 1, h_2 = r, h_3 = r \sin \theta$
(iii) $h_1 = h_2 = h_3 = 1$ (iv) $h_1 = 1, h_2 = r \sin \theta, h_3 = r \cos \theta$
- The position vector $r = \hat{i} x + \hat{j} y + \hat{k} z$ in spherical polar co-ordinates.....
(i) $r \hat{u}_r$ (ii) $r \hat{u}_\theta$ (iii) $\theta \hat{u}_\psi$ (iv) $r \hat{u}_\phi$
- The product of two unitary matrices A and B is.....
(i) unitary (ii) symmetric
(iii) anti-symmetric (iv) orthogonal
- The eigen values of an orthogonal matrix are.....
(i) zero (ii) imaginary (iii) real (iv) a real number
- The Fourier sine transform of function $f(x) = e^{-ax}$ is.....
(i) $\frac{s}{a^2+s^2}$ (ii) $\frac{a}{a^2+s^2}$ (iii) $\frac{s \sin ax}{\sqrt{a^2+s^2}}$ (iv) $\frac{a \sin ax}{\sqrt{a^2+s^2}}$
- The Laplace transform of $t + t^2$ is.....
(i) $\frac{1}{s} + \frac{1}{s^2}$ (ii) $\frac{1}{s^2} + \frac{1}{s^2}$ (iii) $\frac{1}{s^2} + \frac{2}{s^3}$ (iv) $\frac{2}{s^2} + \frac{3}{s^3}$
- The Cauchy integral theorem states that if $f(z)$ is analytic in a simply connected of domain D, then $\int_C f(z) = 0$ on every simply closed path C in domain D. The condition of analyticity in this theorem is.....
(i) necessary (ii) sufficient
(iii) necessary and sufficient (iv) arbitrary
- A function that is analytic at all points of the z-plane and finite at infinity.
(i) must have a singularity (ii) must be zero
(iii) must be a constant (iv) cannot exist

SECTION - B (35 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 7 = 35)

11. (a) Prove that $\int_S \nabla_\phi \times \nabla_\psi \, ds = 0$.

(OR)

- (b) Explain the rank of tensor.

Cont...

12. (a) Explain orthogonal curvilinear co-ordinates.
(OR)
(b) Discuss about cylindrical co-ordinates as a special curvilinear system.
13. (a) Explain orthogonal transformation.
(OR)
(b) Determine the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$
14. (a) Prove that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$, using the Fourier's series.
(OR)
(b) Find the Laplace transform of $\sin at$.
15. (a) Show that the function $f(x) = \sqrt{(|xy|)}$ is not regular at the origin although the Cauchy- Riemann equations are satisfied at that point.
(OR)
(b) Explain the Liouville's theorem.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16. Define a tensor. Prove that the Kronecker symbol δ_i^k is a tensor where components are the same in every co-ordinate system.
17. Explain gradient, divergence and curl in curvilinear co-ordinates.
18. (a) Find the eigen value and corresponding orthogonal vector of the following matrix
- $$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$
- (b) Diagonalize the following matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$
19. Discuss the role of Fourier's series in mathematical physics.
20. Outline the Cauchy- Riemann condition for a function $f(z)$ to be analytic function.

Z-Z-Z

END