18ELU06

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Second Semester)

Branch – **ELECTRONICS**

MATHEMATICS -II

Time: Three Hours | Maximum: 50 Marks

SECTION - A (5 marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. What is the Fourier coefficients a_0 for the function $f(x)$ defined in the interval $(0, 2\pi)$?
   i) $\frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\,dx$ ii) $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$
   iii) $\int_{0}^{2\pi} f(x)\,dx$ iv) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$

2. Which is the Clairaut's form of the first order non-linear partial differential equations?
   i) $F(p,q) = 0$ ii) $z = px + qy + f(p,q)$
   iii) $F(z,p,q) = 0$ iv) $F_1(x,p) = F_2(y,q)$

3. Find $L[\sinh at]$
   i) $\frac{a}{s^2+a^2}$ ii) $\frac{s}{s^2-a^2}$
   iii) $\frac{a}{s^2-a^2}$ iv) $\frac{s}{s^2+a^2}$

4. A vector F is said to be irrational if its curl is ______
   i) 1 ii) 0
   iii) ∞ iv) - 1

5. Green's theorem states that $\int_c (u\,dx + v\,dy) =$ ______
   i) $\iint_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy$ ii) $\iint_R \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dx\,dy$
   iii) $\iint_R \left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right) dx\,dy$ iv) $\iint_R \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) dx\,dy$

SECTION - B (15 marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 3 = 15)

6. a) Show that the Fourier series for $f(x) = x, -\pi < x < \pi$ is given by
   $f(x) = 2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$.

   OR

   b) Define half-range cosine and sine series for the function $f(x)$ defined over the interval $[0, l]$.

7. a) Form the partial differential equation by eliminating the arbitrary constants from
   $z = ax + by + a^2 + b^2$.

   OR

   b) Solve $p^2 + q^2 = 4$.

8. a) Find $L[(t+1)^2]$.

   OR

   b) Find $L(e^{-3t}\sin^2 t)$.

9. a) Find the directional derivative of $f = xyz$ at $(1,1,1)$ in the direction $\vec{i} + \vec{j} + \vec{z}$

   OR

   b) Find $div\ curl\ \vec{F}$ where $\vec{F} = x^2 y\,\vec{i} + xz\vec{j} + 2yz\vec{k}$.

10. a) If $\vec{F} = x^2 i + y^2 j$, evaluate $\int \vec{F} d\vec{r}$ along the line $y = x$ from $(0,0)$ to $(1,1)$

    OR

    b) Show that $\iint_s curl\,\vec{F}.\hat{n}\,ds = 0$, where s is any closed surface.

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SECTION - C (30 marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 6 = 30)

11. a) Obtain the Fourier series of period 2π for the function $f(x) = x^2$ in $(-\pi, \pi)$. Specify the sum of the series at the end points $x = -\pi, \pi$. Deduce the sum of the series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

OR

b). Obtain the half-range sine series of the function $f(x) = kx(x - l)$ in $0 \leq x \leq l$.

12. a) Solve $3p^2 - 2q^2 = 4pq$.

OR

b) Solve $z = p^2 + q^2$.

13. a) Find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$.

OR

b) Solve $y'' + 2y' - 3y = \sin t$ given $y = 0, y'(0) = 0$ when $t = 0$.

14. a) Find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$ of the vector point function $\vec{F} = xz^3\vec{i} - 2x^2yz\vec{j} + 2yz^4\vec{k}$ at the point $(1, -1, 1)$.

OR

b). Find the value of the constant a, b, c so that the vector $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational.

15. a) Verify Gauss Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ and s is the surface of the rectangular parallelepiped bounded by $x = 0, x = a, y = 0, y = b, z = 0, z = c$.

OR

b) Verify Stoke's theorem for a vector field defined by $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region in the XoY plane bounded by the lines $x = 0, x = a, y = 0$ and $y = b$.

Z-Z-Z END