20SSP24

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2022
(Fourth Semester)

Branch – **SOFTWARE SYSTEMS (Five year Integrated)**

LINEAR ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. A system of linear equations is consistent if it has either one solution or --------- solutions.
   (i)Bounded (ii) infinitely many (iii) unbounded (iv) finitely many
2. In echelon form, the leading entry in each non zero row is ----------
   (i) 0 (ii) 1 (iii) 2 (iv) 3
3. If A is a 7X9 matrix with a two – dimensional null space, what is the rank of A?
   (i) 5 (ii) 6 (iii) 7 (iv) 8
4. If $v_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ 7 \\ -9 \end{bmatrix}$ *then* is $\{v_1, v_2\}$ basis for R^3 ?
   (i) yes (ii) No (iii) neither basis (iv)undetermined
5. Let T: $R^n \rightarrow R^m$ be a mapping. The set of all elements in R^n is called ------
   (i) domain (ii) co-domain (iii) range (iv) kernel
6. Let T: $R^n \rightarrow R^m$ be a mapping. In which, the set of all images T(x) is called the______
   (i) domain (ii) co-domain (iii) range (iv) kernel
7. If $< u,v > = 0$ then the two vectors u and v in R^n is called------
   (i) parallel (ii) orthogonal (iii) collinear (iv) perpendicular
8. Which one of the following is correct?
   (i) $\|cv\| = |c|\|v\|$ (ii) $\|cv\| = \|c\|\|v\|$
   (iii) $\|cv\| = |c||v|$ (iv) $\|cv\| = |cv|$
9. Let A and B be nxn matrices, A is invertible if and only if---------
   (i)det A = 0 (ii) det A > 0 (iii) det A < 0 (iv) det A $\neq$ 0
10. det(AB) = --------
    (i)(det A).(det.B) (ii)$(AB)^{-1}$ (iii) $A^T B^T$ (iv) $A^{-1}B^{-1}$

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11. (a) Row reduce the matrix A below to echelon form, and locate the pivot column of A,

$$\text{where } A = \begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}$$

OR

(b) Determine if the following homogeneous system has nontrivial solution. Then describe the solution set.

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 &= 0 \\ -3x_1 - 2x_2 + 4x_3 &= 0 \\ 6x_1 + x_2 - 8x_3 &= 0 \end{aligned}$$

Cont...

12.(a). Find a spanning set for the null space of the matrix $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

OR

(b).Find a matrix A such that W = col A. where $W = \left\{ \begin{bmatrix} 6a-b \\ a+b \\ -7a \end{bmatrix} : a, b \ in \ R \right\}$

13.(a).Define dilation transformation. Find the standard matrix A for the T(x) = 3x for x in R^2.

OR

(b). Explain the shear transformation.

14. (a) If v = (1, -2, 2, 0) then find a unit vector u in the same direction as v.

OR

(b).If W is the subspace of R^2 spanned by x=(2/3,1) then find a unit vector z is a basis for W.

15. (a). If $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}, P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$ then find A^k when $A = PDP^{-1}$.

OR

(b). Prove that an nxn matrix with n distinct eigenvalues is diagonalizable.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16. (a) If $a_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, a_2 = \begin{bmatrix} 5 \\ -13 \\ -3 \end{bmatrix}$ and $b = \begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}$. Then show that span $\{a_1, a_2\}$ is a plane passes through the origin in R^3. Is b in that plane?

OR

(b) Determine the existence and uniqueness of the solutions of the system.

$3x_2 - 6x_3 + 6x_4 + 4x_5 = -5$

$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$

$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15$

17 (a) Show that the set H of all points in R^2 of the form (3s,2+5s) is not a vector space by showing that it is not closed under scalar multiplication.

OR

(b) .State and prove the spanning set theorem.

18(a) Let $T: R^n \to R^m$ be a linear transformation. Then prove that T is one- to-one if and only if the equation T(x) = 0 has only the trivial solution.

OR

(b) Let $T: R^n \to R^m$ be a linear transformation and let A be the standard matrix for T. Then prove that T maps R^n onto R^m if and only if the columns of A span R^m.

19(a) State and prove the QR factorization theorem.

OR

(b) Find a least – squares solution of Ax = b for $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}$.

20(a) State and prove the diagonalization theorem.

OR

(b) Diagonalize the following matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

Z-Z-Z END