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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2022
(Third Semester)

Branch – **COMPUTER SCIENCE WITH DATA ANALYTICS**

LINEAR ALGEBRA

Time: 3 Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. A 1-1 and onto linear transformation is called ------------------.
   (i) Epimorphism (ii) Monomorphism (iii) Homomorphism (iv) Isomorphism
2. The dimension of a vector space C over R is ---------------.
   (i) 2 (ii) 3 (iii) 1 (iv) 5
3. If x and y are orthogonal iff ------------------.
   (i) $<x, y>=0$ (ii) $<x, y>=1$ (iii) $x=y$ (iv) $x=0$
4. A square matrix A is said to be idempotent if A^2 = -------- .
   (i) $\bar{A}$ (ii) A (iii) $-A^T$ (iv) $-\bar{A}$
5. The characteristic roots of skew hermitian matrix are all -----------
   (i) Imaginary (ii) Real (iii) Positive (iv) Negative

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. (a) Prove that the intersection of two sub-spaces of a vector space is a subspace.

(OR)

(b) Let $S=\{v_1, v_2, v_3, \ldots, v_n\}$ be a linearly dependent set of vectors in V iff there exists a Vector $v_k \in S$ such that v_k is a linear combination of the preceding vectors $v_1, v_2, v_3, \ldots, v_{k-1}$.

7. (a) Let $S=\{v_1, v_2, v_3, \ldots, v_n\}$ be a linearly independent set of vectors in V iff there exists a vector $v_k \in S$ such that v_k is a linear combination of the preceding vectors $v_1, v_2, v_3, \ldots, v_{k-1}$.

(OR)

(b) Let V and W be two finite dimensional vector spaces over a field F.Let dim V=m and dim W=n.Then prove that L(V,W) is a vector space of dimension mn over F.

8. (a) Let V be the vector space of polynomials with inner product given by

$$<f, g> = \int_0^1 f(t)g(t)\,dt \quad , f(t) = t+2 \text{ and } g(t) = t^2-2t-3$$

Find (i) $<f,g>$ (ii) $\|f\|$.

(OR)

(b) Let W_1 and W_2 be subspaces of a finite dimensional inner product space.Then prove that

(i) $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$

(ii) $(W_1 \cap W_2)^{\perp} = W_1^{\perp} + W_2^{\perp}$

9. (a) Prove that any square matrix A can be expressed *uniquely* as the sum of a symmetric matrix and a skew symmetric matrix.

(OR)

Cont...

9. (b) Reduce the matrix $A=\begin{pmatrix}1 & 2 & -1\\ 1 & 1 & 2\\ 2 & 4 & -2\end{pmatrix}$

10. (a) State and prove Cayley Hamilton theorem.

(or)

(b) Prove that the characteristic roots of a Hermitian matrix are all real.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. (a) State and prove Fundamental theorem of homomorphism in Vector spaces.

OR

(b) Let V be a vector space over a field F and S be any non-empty subset of V. Then prove the following

(i) L(S) is a subspace of V.

(ii) $S \subseteq L(S)$.

(iii) L(S) is the smallest subspace of V containing S.

12. (a) Let V be a vector space over a field F. Let W be a subspace of V. Then prove the following

$(i)\ \dim W \leq \dim V \quad (ii)\ \dim \frac{V}{W} = \dim V - \dim W.$

OR

(b) Let V be a finite dimensional vector space over a field F. Let A and B be subspaces of V. Then prove that dim(A+B)=dim A + dim B - dim($A \cap B$).

13. (a) Prove that every finite dimensional inner product space has an orthonormal basis.

OR

(b) Prove the norm defined in an inner product space V has the following properties.

i) $|\langle x, y\rangle| \leq \|x\|\,\|y\|$

ii) $\|x + y\| \leq \|x\| + \|y\|$

14. (a) If $A=\begin{pmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{pmatrix}$ show that $A^3-6A^2+7A+2I=0$.

OR

(b) Compute the inverse of the matrix $\begin{pmatrix}1 & 2 & 3\\ 0 & -1 & 4\\ -2 & 2 & 1\end{pmatrix}$

15. (a) Show that the equations

$x + y + z = 6$

$x + 2y + 3z = 14$

$x + 4y + 7z = 30$

are consistent and solve them.

OR

(b) Find the eigen values and eigen vectors of the matrix $A=\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$.

Z-Z-Z END