20MAU14

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Fifth Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 For any set A , A is at most countable if A is ________
(a) finite (b) countable
(c) finite and countable (d) finite or countable

2 The set $P = \bigcap_{n=1}^{\infty} E_n$ is called the __________ set
(a) Cantor (b) countable
(c) closed (d) compact

3 A metric space in which every Cauchy sequence converges is said to be ________
(a) complete (b) closed
(c) convergent (d) Cauchy

4 A mapping f of a set E into R^k is said to be bounded if there is a real number M such that _________ for all $x \in E$
(a) $|f(x)| < M$ (b) $|f(x)| \le M$
(c) $|f(x)| > M$ (d) $|f(x)| \ge M$

5 Suppose f is differentiable in (a, b) and $f(x) \ge 0$, for all $x \in (a,b)$ then f is _____ in (a, b)
(a) constant (b) monotonically increasing
(c) monotonically decreasing (d) not continuous

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 (a) Let { E_n }, n=1,2,3,... be a sequence of countable sets and $S = \bigcup_{n=1}^{\infty} E_n$. Then prove that S is countable.

(OR)

(b) If P is a limit point of a set E, then prove that every neighborhood of P contains infinitely many points of E.

7 (a) If E is an infinite subset of a compact set K, then prove that E has a limit point in K.

(OR)

(b) A subset E of the real line R^1 is connected if and only if it has the following property: If $x \in E$, $y \in E$ and x < z < y, then prove that $z \in E$

8 (a) Suppose $\{S_n\}$ is monotonic . Then prove that $\{S_n\}$ converges if and only if it is bounded.

(OR)

(b) State and prove Root Test.

9 (a) Suppose f is continuous mapping of a compact metric space X into metric space Y. Then prove that $f(X)$ is compact.

(OR)

Cont...

9 (b) Suppose f is a continuous 1-1 mapping of a compact metric space X onto metric space Y. Then the inverse mapping f^{-1} defined on Y by $f^{-1}(f(X)) = x(x \in X)$ is continuous mapping Y onto X.

10 (a) If f and g are continuous real functions in[a, b] which are differentiable in (a,b), then prove that there is a point $x \in (a,b)$ at which $[f(b)-f(a)]g'(x) = [g(b)-g(a)]f'(x)$

(OR)

(b) Suppose f is a real differentiable function on [a, b] and suppose $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 (a) (i) Let A be the set of all sequences whose elements are the digits 0 and 1. Then prove that A is uncountable.

(ii) Define Metric space.

(OR)

(b) (i) Prove that every neighborhood is an open set.

(ii) Suppose $Y \subset X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

12 (a) Prove that every k-cell is compact.

(OR)

(b) State and prove Weierstrass theorem.

13 (a) Suppose $\{s_n\}, \{t_n\}$ are complex sequences , and $\lim_{n\to\infty} s_n = s$, $\lim_{n\to\infty} t_n = t$. Then Prove that

(i) $\lim_{n\to\infty}(s_n + t_n) = s + t$

(ii) $\lim_{n\to\infty} cs_n = cs$, $\lim_{n\to\infty}(c + s_n) = c + s$, for any number c

(iii) $\lim_{n\to\infty} s_n t_n = st$

(iv) $\lim_{n\to\infty} \frac{1}{s_n} = \frac{1}{s}$, provided $s_n \neq 0$ (n=1,2,3...) and $s \neq 0$.

(OR)

(b) (i) Define a power series.

(ii) State and prove Ratio test.

14 (a) Prove that a mapping f of a metric space X into metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

(OR)

(b) Let f be a continuous mapping of a compact metric space X into metric space Y. Then prove that f uniformly continuous on X.

15 (a) State and prove L' Hospital's rule.

(OR)

(b) State and prove Taylor's theorem.

Z-Z-Z END