

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Third Semester)

Branch – MATHEMATICS

PARTIAL DIFFERENTIAL EQUATIONS & FOURIER TRANSFORMS

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(5 x 1 = 5)

- 1 The partial differential equation by eliminating the constants from $z = ax + by + ab$ is _____.
 (i) $z = px + qy + pq$ (ii) $z = px + qy - pq$
 (iii) $z = px + qy$ (iv) $z = px - qy$
- 2 If $u_{xx} + x^2 u_{yy} = 0$ then the given PDE equation is _____.
 (i) Parabolic (ii) Elliptic
 (iii) Hyperbolic (iv) Mixed type
- 3 If $f(x)$ is an even function then _____.
 (i) $f(-x) = -f(x)$ (ii) $f(-x) = f(x)$
 (iii) $f(-x) = -1/f(x)$ (iv) 0
- 4 The Fourier cosine transform of $e^{-x^2/2} =$ _____.
 (i) $e^{-w^2/2}$ (ii) $-e^{-w^2/2}$
 (iii) $e^{w^2/2}$ (iv) $-e^{w^2/2}$
- 5 If $\partial u / \partial t = 0$ then the heat equation becomes _____ equation.
 (i) Diffusion (ii) Laplace
 (iii) Wave (iv) Poisson

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 3 = 15)

- 6 a Form the partial differential equation by eliminating the arbitrary function from $f(x + y + z, x^2 + y^2 + z^2) = 0$.
 OR
 b Find the complete integral of $p^2 + q^2 = x + y$.
- 7 a Find the characteristics of the equation $u_{xx} + 2u_{xy} + \sin^2(x)u_{yy}u_y = 0$.
 OR
 b Solve $(D^2 + 5D + 6)u = e^x$.
- 8 a Find the Fourier coefficient a_n of the periodic function $f(x) = |x|$ for $-\pi < x < \pi$.
 OR
 b Find the Fourier series of the function

$$f(x) = \begin{cases} -k & \text{if } -2 < x < 0 \\ k & \text{if } 0 < x < 2 \end{cases} \quad \text{and } L = 2.$$

Cont...

- 9 a Consider the periodic rectangular wave $f_L(x)$ of period $2L > 2$ given by

$$f(x) = \begin{cases} 0 & \text{if } -L < x < -1 \\ 1 & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < L \end{cases}$$

OR

- b Find the Fourier transform of xe^{-x^2} .
- 10 a Find the temperature $u(x, t)$ in a laterally insulated copper bar 80 cm long if the initial temperature is $100\sin(\pi x/80)^\circ\text{C}$ and the ends are kept at 0°C . How long will take for the maximum temperature in the bar to drop to 50°C ? Calculate the physical data for copper: density 8.92 g/cm^3 , specific heat $0.092\text{ cal/(g}^\circ\text{C)}$, thermal conductivity $0.95\text{ cal/(cm sec}^\circ\text{C)}$.

OR

- b Solve the heat equation $\frac{\partial u}{\partial t} = c^2 \partial^2 u / \partial x^2$, $u(x, 0) = f(x)$, $(-\infty < x < \infty)$, where $f(x)$ is the given initial temperature of the bar.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 x 6 = 30)

- 11 a Find the general integral of the linear PDE $pz - qz = z^2 + (x + y)^2$.
- OR
- b Find the complete integral of the PDE $z^2 = pqxy$.
- 12 a Reduce the equation $(1 + x^2)u_{xx} + (1 + y^2)u_{yy} + xu_x + yu_y = 0$ to a canonical form.
- OR
- b Solve $(D^2 + 16)u = e^{-3x} + \cos 4x$.
- 13 a Find the Fourier series of the given function $f(x) = x^2$, $(-\pi < x < \pi)$ which is assumed to have the period 2π .
- OR
- b Find the Fourier series of the given function $f(x) = x + \pi$, $(-\pi < x < \pi)$ and $f(x + 2\pi) = f(x)$.
- 14 a Find the Fourier cosine transform of $\mathcal{F}_c(e^{-ax})$ of $f(x) = e^{-ax}$, where $a > 0$.
- b Find the Fourier transform of $f(x) = 1$ if $|x| < 1$ and $f(x) = 0$ otherwise.
- 15 a Find the temperature in a laterally insulated bar of length L assuming that whose ends are insulated,

$$f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}$$

OR

- b Solve the temperature in the infinite bar if the initial temperature is given below by the method of convolution.

$$f(x) = \begin{cases} U_0 = \text{const} & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$