22MAU410

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Fourth Semester)

Branch – **MATHEMATICS**

MODERN ALGEBRA

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. If p is a prime number and a is any integer, then $a^P \equiv a \ mod p$ this theorem stated by
   i) Euler ii) Lagrange's iii) Modulo iv) Fermat

2. If G is a group, N a normal subgroup of G, then $G|N$ is also a group. It is called the --------of G by N.
   i) Normal subgroup ii) Abelian group iii) Quotient group iv) Subgroup

3. Any group of order p has __________ subgroups.
   i) P ii) P+1 iii) P-1 iv) trivial

4. A------- is a commutative division ring.
   i) Field ii) Commutative ring.
   iii) Division ring iv) Integral domain

5. Every -------- can be imbedded in a field.
   i) Field ii) Commutative ring iii) Division ring iv) Integral domain

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 .a) Prove that If H is a nonempty finite subset of a group G and H is closed under multiplication, then H is a subgroup of G.

(OR)

b) Prove that If G is a group, then For all $a, b \in G, (a \cdot b)^{-1} = b^{-1}.a^{-1}$.

7. a) Prove that If H and K are finite subgroups of G of orders o(H) and o(K), respectively, then $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$

(OR)

b) Prove that If G is a finite group and N is a normal subgroup of G, then $o(G/N) = o(G)/o(N)$.

8.a) Prove that every permutation is the product of its cycles.

(OR)

b) Prove that let G be a group and ϕ is an automorphism of G. If $a \in G$, if $o(a) > 0$, then $o(\phi(a)) = o(a)$.

9.a) Prove that a finite integral domain is a field.

(OR)

b) Define (i) zero-divisor (ii)integral domain (iii) field

10.a) Prove that let R be a Euclidean ring. Then every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R.

(OR)

b) Prove that let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then R is a field

Cont...

SECTION -C (30 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Let G be the set of all 2 x 2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are real numbers, such that $ad - bc \neq 0$.
(i) Define group with 2 examples
(ii) Prove that G with matrix multiplication is not abelian?
(OR)

b) (i) Prove that If G is a finite group and a $\in$ G, then $o(a)/\ o(G)$.
(ii) Prove that If G is a finite group and a $\in$ G, then $a^{o(G)} = e$.

12. a) Prove that Let ϕ be a homomorphism of G onto G with kernel K. Then $G/K \approx \bar{G}$.
(OR)
b) State and Prove Sylow's theorem for Abelian Groups.

13. a) Prove that Every group is isomorphic to a subgroup of A(S) for some appropriate S.
(OR)
b) Prove that if G is a group, then $\mathcal{A}(G)$, the set of automorphisms of G, is also a group.

14. a) Prove that J_p is a field. Given J_p is the ring of integers mod p, p is a prime.
(OR)
b) If R is a ring, thenfor all a, b $\in$ R
1. $a0 = 0a = 0$.
2. $a(-b) = (-a)b = -(ab)$.
3. $(-a)(-b) = ab$. If in addition, R has a unit element 1, then
4. $(-1)a = -a$.
5. $(-1)(-1) = 1$.

15. a) Prove that J[i] is a Euclidean ring.
(OR)
b) State and Prove Unique Factorization Theorem.

Z-Z-Z END