PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Sixth Semester)

Branch – **MATHEMATICS**

LINEAR ALGEBRA

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. If the matrix $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then $x =$ _____.
   a) 3 b) 5 c) 2 d) 4

2. $F^{(n)}$ is isomorphic $F^{(m)}$ if and only if _____.
   a) $n < m$ b) $n > m$ c) $n \neq m$ d) $n = m$

3. Let $V = \mathbb{R}^2$ with inner product defined by $< u, v >= u_1v_1 + u_2v_2$, then $||(1,2)|| =$ ___.
   a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\sqrt{4}$ d) $\sqrt{5}$

4. Find the value of k such that the rank of a matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & k \end{bmatrix}$ is 1
   a) 8 b) 16 c) 12 d) 9

5. The kernel of a matrix transformation $T_A: \mathbb{R}^n \to \mathbb{R}^m$ is a subspace of
   a) $\mathbb{R}^n$ b) $\mathbb{R}^m$ c) $\mathbb{R}^{n-m}$ d) $\mathbb{R}^{n+m}$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) Prove that given A is a Hermitian Matrix, then iA is a skew Hermitian matrix.

OR

   b) Show that the matrix $A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

7. a) If U and W are subspaces of V, prove that $U + W = \{v \in V | v = u + w, u \in U, w \in W\}$ is a subspace of V.

OR

   b) If F is the field of real numbers, prove that vectors $(1,1,0,0)$, $(0,1,-1,0)$ and $(0,0,0,3)$ in $F^{(4)}$ are linearly independent over F.

8. a) Prove that $A(W)$ is a subspace of $\hat{V}$.

OR

   b) State and prove Schwarz inequality.

9. a) Show that the matrices A and $P^{-1}AP$ have the same characteristic roots.

OR

   b) Prove that the characteristic roots of a Hermitian matrix are all real.

Cont...

10. a) Compute the matrix product $\begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ 3 & 4 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & -1 & -1 \end{pmatrix}$.

OR

b) Prove that if $T \in A(V)$ and if $\dim_F V = n$ then T can have atmost n distinct characteristic roots in F.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 x 6 = 30)

11. a) If A be $m \times n$ matrix and B be $n \times p$ matrix then prove that $(AB)^T = B^T A^T$.

OR

b) Prove that for given A and B be symmetric matrices of order n then AB+BA is symmetric and AB is symmetric if $AB = BA$.

12. a) If V is the internal direct sum of $U_1, \ldots, U_n$ then prove that V is isomorphic to the external direct sum of $U_1, \ldots, U_n$.

OR

b) Prove that if $v_1, v_2, \ldots, v_n$ are in V then either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1, v_2, \ldots, v_{k-1}$.

13. a) Prove that let V and W be vector spaces over the field F then Hom (V, W) is a vector space over F.

OR

b) Prove that for given V is a finite-dimensional inner product space and W is a subspace of V, then $V = W + W^T$.

14. a) Prove that for given A and B are two square matrices, then the matrices AB and BA have the same characteristic roots.

OR

b) Prove that the Matrix $A = \begin{bmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ a satisfies Cayley-Hamilton theorem, also find the matrix A^{-1}.

15. a) Given V is a vector space over F then for $S, T \in A(V)$. Prove that.

1. $r(ST) \leq r(T)$
2. $r(TS) \leq r(T)$
3. $r(ST) \leq r(TS) = r(T)$ for S regular in $A(V)$.

OR

b) Prove that for given $T, S \in A(V)$ and if S is regular, then T and STS^{-1} have the same minimal polynomial.

Z-Z-Z END