20MAU23A

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Sixth Semester)

Branch – **MATHEMATICS**

DISCIPLINE SPECIFIC ELECTIVE – II: NUMERICAL METHODS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. If $g \in C[a,b]$ and $g(x) \in [a,b]$ for all $x \in [a,b]$ then ____.
   (a) g has exactly one fixed point in [a,b]
   (b) g has atleast one fixed point in [a,b]
   (c) g has no fixed point in [a,b]
   (d) $g'(x)$ exists on [a,b]

2. If $x_0, x_1, x_2, \dots x_n$ are n+1 distinct numbers and f is a function whose values are given at these numbers, then a unique polynomial P(x) of degree______ exists.
   (a) n+1 (b) n+1 or less (c) n (d) n or less

3. The midpoint rule approximation of $\int_{0.5}^{1} x^4 dx =$______.
   (a) 0.528123 (b) 0.158203 (c) 0.281253 (d) 0.825123

4. Which of the following methods give more accurate results for initial value problems?
   (a) Taylor's method (b) Runge-Kutta method of order two
   (c) Euler's method (d) Runge-Kutta method of order four.

5. Which of the following methods is employed for solving a system of linear equations?
   (a) Neville's (b) Newton's (c) Gauss Siedel (d) Simpson's rule

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. (a) Determine the number of iterations necessary to solve $f(x) = x^3 + 4x^2 - 10 = 0$ with accuracy 10^{-3} using $a_1 = 1$ and $b_1 = 2$.

(OR)

(b) Show that $g(x) = \frac{(x^2 - 1)}{3}$ has a unique fixed point on the interval [-1,1].

7. (a) Determine the second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$ using the numbers $x_0 = 2$, $x_1 = 2.75$ and $x_2 = 4$.

(OR)

(b) The values of f(x) = ln x accurate to the places are given in the following table.

i	x_i	ln x_i
0	2.0	0.6931
1	2.2	0.7885
2	2.3	0.8329

Apply Neville's method and four digit rounding arithmetic to approximate f(2.1) = ln 2.1 by completing the Neville's table.

8. (a) Apply the Simpson's rule and find the value of $\int_0^2 f(x)dx$ when $f(x) = \sqrt{1+x^2}$. Compare the result with exact value of the integral.

(OR)

(b) Use the composite Trapezoidal rule and find the value of $\int_0^{3\pi/8} tanx\, dx$ with n = 8.

9. (a) Show that the initial value problem $\frac{dy}{dx} = y - t^2 + 1, 0 \le t \le 2$, y(0) = 0.5 is well posed on $D = \{(t,y) \mid 0 \le t \le 2 \text{ and } -\infty < y < \infty\}$.

(OR)

(b) Apply Euler's method to approximate the solution for the initial value problem $y' = t\, e^{3t} - 2y, 0 \le t \le 1$, y(0) = 0 with h = 0.5.

Cont...

10. (a) Apply Gaussian elimination method and solve the following system of equations.

$x_1 + x_2 + x_3 = 4$
$2x_1 + 2x_2 + x_3 = 4$
$x_1 + x_2 + 2x_3 = 6$

(OR)

(b) If the spectral radius satisfies $\rho(T) < 1$, then show that $(I - T)^{-1}$ exists and $(I - T)^{-1} = I + T + T^2 + \ldots = \sum_{j=0}^{\infty} T^j$

SECTION – C **(5 × 6 = 30 MARKS)**

ANSWER ALL QUESTIONS

ALL questions carry **EQUAL** marks

11. (a) Show that if A is any positive number, then the sequence defined by $x_n = \frac{1}{2}x_{n-1} + \frac{A}{2x_{n-1}}$, for $n \geq 1$, converges to $\sqrt{A}$ whenever $x_0 > 0$.

(OR)

(b) Apply Secant method to find a solution to $x - \cos x = 0$ in the interval $\left[0, \frac{\pi}{2}\right]$ that is accurate to within 10^{-4}.

12. (a) Apply the Newton's divided difference formula to construct interpolating polynomials of degree one, two ,three and find the value of f(8.4) using each of the polynomials given f(8.1) = 16.94410, f(8.3) = 17.56492, f(8.6) = 18.50515, f(8.7) = 18.82091.

(OR)

(b) For the following table of data apply Newton's backward difference formula and determine the value of f(2.0).

x	f(x)
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

13. (a) Derive the three-point formulas for numerical differentiation.

(OR)

(b) Use Romberg integration to compute $R_{3,3}$ for the integral $\int_3^{3.5} \frac{x}{\sqrt{x^2-4}}\,dx$.

14. (a) Suppose f is continuous and satisfies a Lipchitz condition with constant L on $D = \{(t,y) \mid a \leq t \leq b \text{ and } -\infty < y < \infty\}$ and that a constant M exists with $|y''(t)| \leq M$ for all $t \in [a,b]$, where y(t) denotes the unique solution to the initial value problem $y' = f(t,y)$, $a \leq t \leq b$, $y(a) = \alpha$. Let $w_0, w_1, w_2, \ldots w_N$ be the approximations generated by Euler's method for some positive integer N. Then show that for each i = 0, 1, 2,..., N,

$$|y(t_i) - w_i| \leq \frac{hM}{2L}\left[e^{L(t_i-a)} - 1\right].$$

(OR)

(b) Use the Modified Euler method to approximate the solution for the initial value problem $y' = \cos 2t + \sin 3t$, $0 \leq t \leq 1$, $y(0) = 1$ with h = 0.25 and actual solution $y(t) = \frac{1}{2}\sin 2t - \frac{1}{3}\cos 3t + \frac{4}{3}$.

15. (a) Solve the following system of equations by Gaussian elimination method.

$x_1 - x_2 + 2x_3 - x_4 = -8$
$2x_1 - 2x_2 + 3x_3 - 3x_4 = -20$
$x_1 + x_2 + x_3 = -2$
$x_1 - x_2 + 4x_3 + 3x_4 = 4$

(OR)

(b) Solve the following system of equations by Gauss Seidel-iterative technique.

$10x_1 - x_2 + 2x_3 = 6$
$-x_1 + 11x_2 - x_3 + 3x_4 = 25$
$2x_1 - x_2 + 10x_3 - x_4 = -11$
$3x_2 - x_3 + 8x_4 = 15$

Z-Z-Z END