PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Second Semester)

Branch – **MATHEMATICS**

DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks

$(10 \times 1 = 10)$

Module No.	Question No.	Question	K Level	CO
1	1	Torricelli's law is given by _____. a) $\frac{dt}{dV} = -k\sqrt{y}$ b) $\frac{dV}{dt} = k\sqrt{y}$ c) $\frac{dV}{dt} = -k\sqrt{y}$ d) $\frac{dt}{dV} = k\sqrt{y}$	K1	CO1
	2	The integrating factor of the linear first order differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is _____. a) $e^{\int P(x)dx}$ b) $e^{-\int P(x)dx}$ c) $e^{-\int Q(x)dx}$ d) $e^{\int Q(x)dx}$	K1	CO1
2	3	The general solution of $y'' + y = 0$ is $y(x) =$ _____. a) $c_1 \cos x - c_2 \sin x$ b) $c_1 \cos x + c_2 \sin x$ c) $c_1 \sin x - c_2 \cos x$ d) $c_1 \cos x - c_2 \sec x$	K2	CO2
	4	The Wronskian of (e^x, xe^x) is _____. a) xe^x b) e^x c) e^{2x} d) xe^{2x}	K2	CO2
3	5	If the operational determinant is identically zero, then the system is said to be _____. a) generate b) degenerate c) independent d) dependent	K1	CO3
	6	A linear first-order system is _____ if the functions are all identically zero. a) non homogeneous b) homogeneous c) independent d) dependent	K1	CO3
4	7	Laplace transform of function f(t) is given by _____. a) $F(t) = \int_0^\infty e^{-t} f(t)dt$ b) $F(s) = \int_0^\infty e^{-st} f(t)dt$ c) $f(t) = \int_0^\infty e^{-t} f(t)dt$ d) $f(s) = \int_0^\infty e^{-st} f(t)dt$	K1	CO4
	8	$\Gamma\left(\frac{5}{2}\right) =$ _____. a) 1 b) $\frac{1}{2}\sqrt{\pi}$ c) $\sqrt{\pi}$ d) $\frac{3}{4}\sqrt{\pi}$	K2	CO4
5	9	Which of the following is not a property of Laplace transforms? a) Linearity b) Differentiation property c) Convolution property d) Time-shift property	K1	CO5
	10	$F'(s) =$ _____. a) $L[t\,f(t)]$ b) $L[-\left(\frac{f(t)}{t}\right)]$ c) $L[-t\,f(t)]$ d) $L[-\left(\frac{f(t)}{t}\right)]$	K2	CO5

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks   (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	The population of a colony of bacteria at time t = 0 hours was 1000, and that the population doubled after 1 hour. Predict the number of bacteria in the population after 1½ hours.	K3	CO1
		(OR)		
	11.b.	Solve the initial value problem $x^2 \dfrac{dy}{dx} + xy = \sin x$, $y(1) = y_0$.		
2	12.a.	Evaluate the initial value problem $y'' - 2y' + y = 0$ given $y(0) = 3$, $y'(0) = 1$.	K4	CO2
		(OR)		
	12.b.	Show that the three solutions $y_1(x) = x$; $y_2(x) = x \ln x$ and $y_3(x) = x^2$ of the third-order equation $x^3 y^{(3)} - x^2 y'' + 2xy' - 2y = 0$ are linearly independent on the open interval x>0. Hence find its particular solution that satisfies the initial conditions $y(1) = 3$; $y'(1) = 2$; $y''(1) = 1$.		
3	13.a.	Obtain the general solution of $x' = y$ and $y' = 2x + y$.	K3	CO3
		(OR)		
	13.b.	Derive a general solution of the system $(D - 4)x + 3y = 0$, $-6x + (D + 7)y = 0$.		
4	14.a.	Evaluate $L(\cos h^2 t)$.	K4	CO4
		(OR)		
	14.b.	Evaluate the inverse Laplace transform of $\dfrac{1}{s^2(s-a)}$.		
5	15.a.	Obtain $L^{-1}[\tan^{-1}\left(\dfrac{1}{s}\right)]$.	K3	CO5
		(OR)		
	15.b.	A mass m = 1 is attached to a spring with constant k = 4; there is no dashpot. The mass is released from rest with x(0) = 3. At the instant $t = 2\pi$ the mass is struck with a hammer, providing an impulse p=8. Determine the motion of the mass.		

SECTION -C (30 Marks)
Answer **ANY THREE** questions
ALL questions carry **EQUAL** Marks $(3 \times 10 = 30)$

Module No.	Question No.	Question	K Level	CO
1	16	Assume that Lake Erie has a volume of 480 km^3 and that its rate of inflow (from Lake Huron) and outflow (to Lake Ontario) are both 350 km^3 per year. Suppose that at the time $t = 0$ (years), the pollutant concentration of Lake Erie caused by past industrial pollution that has now been ordered to cease is five times that of Lake Huron. If the outflow henceforth is perfectly mixed lake water, how long will it take to reduce the pollution concentration in Lake Erie to twice that of Lake Huron?	K4	CO1
2	17	Find the particular solution of $3y'' + y' - 2y = 2 \cos x$.	K3	CO2
3	18	Obtain the particular solution of the system $x' = 4x - 3y$, $y' = 6x - 7y$ that satisfies the initial conditions $x(0) = 2$, $y(0) = -1$.	K3	CO3
4	19	Using Laplace Transform, solve the initial value problem $x'' - x' - 6x = 0$, $x(0) = 2$, $x'(0) = -1$.	K3	CO4
5	20	Consider the RLC circuit with R = 110 Ω, L = 1 H, C = 0:001 F and a battery supplying $E_0 = 90$ V. Initially there is no current in the circuit and no charge on the capacitor. At time $t = 0$ the switch is closed and left closed for 1 second. At time $t = 1$ it is opened and left open thereafter. Calculate the resulting current in the circuit.	K4	CO5

Z-Z-Z   END