PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(First Semester)

Branch – **MATHEMATICS**

DIFFERENTIAL EQUATIONS & LAPLACE TRANSFORM

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 Choose the mathematical model of "the time rate of change of a population P is proportional to the square root of P".
(i) $\frac{dp}{dt} = \sqrt{P}$ (ii) $\frac{dp}{dt} = k\sqrt{P}$ (iii) $\frac{dp}{dt} = P(t)$ (iv) $\frac{dt}{dp} = \sqrt{P}$

2 Which of the following pair of functions are linearly independent on the real line?
(i) sinx and cosx (ii) e^{x} and e^{-2x}
(iii) e^{x} and xe^{x} (iv) All the above

3 A resistor with a resitance of ____
(i) L henries (ii)C farads (iii)R ohms (iv) I amperes

4 Find $\Gamma(1)$=____
(i) 0 (ii)1 (iii) 2 (iv) -1

5 Which of the following is true?
(i) $L[f(t)*g(t)] = L[(f(t)]\cdot L[g(t)]$ (ii) $L[f(t)*g(t)] = L[(f(t)]\pm[g(t)]$
(iii) $L[f(t)*g(t)] = L[(f(t)] - L[g(t)]$ (iv) $L[f(t)*g(t)] = L^{-1}(f(t)] - [g(t)]]$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Solve the differential equation $\frac{dy}{dx} = y^{2}$, y(1)=2

OR

b Solve the initial value problem $\frac{dy}{dx} = 2x+3$, y(1)=2

7 a Calculate general solution of the differential equation $y'' - 2y' + y = 0$ given y(0)=3,y'(0)=1

OR

b Use Wronskian to show that the following functions are linearly independent $y_1(x) = e^{-3x}$, $y_2(x) = \cos 2x$, and $y_3(x) = \sin 2x$.

8 a Find a particular solution of $y'' - 4y = 2e^{3x}$

OR

b Determine a general solution of the system $x' = y, y' = 2x + y$

9 a Calculate (i) $L[3t^{2} + 4t^{3/2}]$ (ii) $L[\cosh kt]$

OR

b Calculate inverse laplace transform of $\frac{s^{2}+1}{s^{3} - 2s^{2} - 8s}$

10 a Calculate convolution of cost and sint.

OR

b Find $L^{-1}\left[\frac{2s}{(s^{2}-1)^{2}}\right]$

Cont...

SECTION -C (30 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Solve the differential equation $\frac{dy}{dx} = (x+y+3)^2$

OR

b Solve the differential equation $(6xy-y)^3\,dx + (4y+3x^2-3xy^2)dy = 0$

12 a Calculate particular solution of the equation $y'' + 4y = 12x$ given y(0)=5, y'(0)=7.

OR

b Solve the initial value problem

$y^3 + 3y'' - 10y' = 0, y(0) = 7, y'(0) = 0, y''(0) = 70$

13 a Find a particular solution of the equation $y'' + y = \tan x$.

OR

b Consider an RLC circuit with R = 50 ohms ,L=0.1 henry and C =$5X10^{-4}$ farad.At time t=0 when both I(0) and Q(0) are zero, the circuit is connected to a 110-V ,60-Hz alternating current generator. Find the current in the circuit and the time lag of the steady periodic current behind the voltage.

14 a Solve the initial value problem $x'' - x' - 6x = 0, x(0) = 2, x'(0) = -1$

OR

b Solve the initial value problem $y'' + 4y' + 4y = t^2, y(0) = y'(0) = 0$

15 a Find (i) $L\left[t^2 \sin kt\right]$ (ii) $L\left[\frac{\sinh t}{t}\right]$

OR

b Solve the initial value problem

$x'' + 4x = 8\delta_{2\pi}(t)$; $x(0) = 3, x'(0) = 0$.

Z-Z-Z END