20MAU13

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Fifth Semester)

Branch – **MATHEMATICS**

ALGEBRA

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 If σ maps S onto S, then σ is _______
(i) one-to-one (ii) onto
(iii) bijective (iv) surjective

2 If $a \in G, a^m = e$, then ______
(i) $O(a)/_m$ (ii) $O(m)/_a$
(iii) $O(a)/_{m+1}$ (iv) $O(m+1)/_a$

3 If H,K are subgroups of the abelian group G, then HK is ___________ of G.
(i) Proper subgroup (ii) subgroup
(iii) normal subgroup (iv) abelian

4 A ring is said to be a _________ if its non zero elements form a group under multiplication.
(i) integral domain (ii) Euclidean ring
(iii) commutative ring (iv) division ring

5 The units in a commutative ring with a unit element form ______.
(i) a group (ii) a subgroup
(iii) an abelian group (iv) a normal group

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a If $\sigma: S \to T, \tau: T \to U$, and $\mu: U \to V$, then $(\sigma \circ \tau)\mu = \sigma \circ (\tau \circ \mu)$.

OR

b Prove that a nonempty subset H of the group G is a subgroup of G iff $a, b \in H \Rightarrow ab \in H$.

7 a Suppose G is a group, N is a normal subgroup of G, define a mapping $\varphi: G \to G/_N, \varphi(x) = Nx, \forall x \in G$. Then prove that φ is a homomorphism of G onto $G/_N$.

OR

b Prove that N is a normal subgroup of G iff $gNg^{-1} = N, \forall g \in G$.

8 a Prove that $\mathcal{J}(G) \cong G/_N$.

OR

b Let G be a group of order 99 and suppose that H is a subgroup of G of order 11, then H is a normal subgroup of G.

9 a If R is a ring, then for all $a, b \in R$, then prove that $(-a)(-b) = ab$.

OR

b If φ is a homomorphism of R into R', then $\varphi(0) = 0$.

Cont...

10 a Prove that a Euclidean ring possesses a unit element.

OR

b Define Integral Domain.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Prove that the relation $a \equiv b \ mod \ H$ is an equivalence relation.

OR

b Show that for all $a \in G, Ha = \{x \in G | a \equiv x \ mod \ H\}$.

12 a If H and K are finite subgroups of G of orders $O(H)$ and $O(K)$, then prove that $O(HK) = \frac{O(H).O(K)}{O(H\cap K)}$.

OR

b If G is abelian of order $O(G)$ and $p^{\alpha}/_{O(G)}, p^{\alpha+1} \nmid O(G)$ then prove that there is a unique subgroup of G of order p^{α}.

13 a State and prove Cayley's theorem.

OR

b Prove that $O(A_n) = \frac{n!}{2}$.

14 a If φ is a homomorphism of R into R' with kernel $I(\varphi)$, then prove that (i) $I(\varphi)$ is a subgroup of R under addition (ii) If $a \in I(\varphi), r \in R$ then both $ar, ra \in I(\varphi)$.

OR

b If R is a ring, then for all $a, b \in R$, then prove that $(i) a0 = 0a = 0$, $(ii) a(-b) = (-a)b = -(ab)$, $(iii)(-1)a = -a$ and $(iv)(-1)(-1) = 1$.

15 a Let R be an Euclidean ring and let A be an ideal of R, then prove that there exists an element $a_0 \in A$ such that A consists exactly of all $a_0 x$ as x ranges over R.

OR

b The ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R iff a_0 is a prime element of R – prove.

Z-Z-Z END