#### PSG COLLEGE OF ARTS & SCIENCE (AUTONOMOUS)

# MSc DEGREE EXAMINATION DECEMBER 2023

(First Semester)

#### Branch - STATISTICS

### ADVANCED PROBABILITY THEORY

Time: Three Hours

Maximum: 75 Marks

### SECTION-A (10 Marks)

Answer ALL questions

|               | <b>ALL</b> questions carry <b>EQUAL</b> marks $(10 \times 1 = 10)$ |  |            |     |  |  |  |
|---------------|--|--|------------|-----|--|--|--|
| Module<br>No. | Question No.   | Question   | K<br>Level | СО  |  |  |  |
| 1             | 1  | For any two events A and B, P(A-B) is equal to a) P(A)-P(B) b) P(B)-P(A) c) P(B)-P(AB) d) P(A)-P(AB)   | K1         | CO1 |  |  |  |
|               | 2  | A probability distribution satisfies the conditions are  a) $f(x) \ge 0$ and $\sum f(x) = 1$ b) $f(x) < 0$ and $\sum f(x) = 1$ c) $f(x) = 0$ and $\sum f(x) = 1$ d) $f(x) \ge 0$ and $\sum f(x) < 1$ | K2         | CO1 |  |  |  |
| 2             | 3  | By uniqueness property the correlation between T1 and T2 is  a) not linear b) linear c) unity d) not unity   | K1         | CO2 |  |  |  |
|               | 4  | Check about nature of continuity of f(x)= x/(x+1) in [0,2]?  a) Not continuous b) continuous but uniformly continuous c) not uniformly continuous d) uniformly continuous                            | K2         | CO2 |  |  |  |
| 3             | 5  | The word 'stochastic' is derived from a) Greek b) Latin c) Rome d) German  | K1         | CO3 |  |  |  |
|               | 6  | Borel- cantelli lemma is also known as a) zero – two law b) zero – one law c) one – zero law d) converse   | K2         | CO3 |  |  |  |
| 4             | 7  | Convergence in the r <sup>th</sup> mean is<br>a) $r \ge 1$ b) $r < 1$<br>c) $r > 1$ d) $r = 1$   | K1         | CO4 |  |  |  |
|               | 8  | $X_n$ convergence to X almost surely, $X_n \rightarrow X$ , if there is a set $B \subset \Omega$ such that $P(B) = a$ a) -1 b) $\infty$ c) 1 d) 0  | K2         | CO4 |  |  |  |
| 5             | 9  | The differential form of Kolmogorov equation is known a) Differential equation b)State equation c) Chain equation d) Master equation   | K1         | CO5 |  |  |  |
|               | 10   | The one step transition probabilities are denoted by  a) $P_{jk}^{(1)}$ b) $P_{jk}^{(m)}$ c) $P_{jj}^{(1)}$ d) $P_{jj}^{(n)}$  | K2         | CO5 |  |  |  |

# SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

 $(5\times7=35)$ 

| Module | Question | 0  | YZ Y    | CO  |
|--------|----------|--|---------|-----|
| No.    | No.      | Question   | K Level | CO  |
| 1      | 11.a.    | If X and Y are independent random variables,<br>then $E(X + Y)=E(X) + E(Y)$  | K1      | GO1 |
|        | (OR)     |  |         | CO1 |
|        | 11.b.    | Write a short note on chebychev's inequality.  |         |     |
| 2      | 12.a.    | If the distribution function of a random variable X is symmetrical about zero, i.e., $1-F(x)=F(-x) \rightarrow f(-x)=f(x)$ , then $\phi_X(t)$ is real valued and even function of t. | K2      | CO2 |
|        |          | (OR)   |         |     |
|        | 12.b.    | Explain uniqueness theorem with an example.  |         |     |
| 3      | 13.a.    | What is the probability that in a sequence of bernoulli trails with probability of success p for each trail, the pattern SFS appears infinitely often?                               | K1      | CO3 |
|        | (OR)     |  |         |     |
|        | 13.b.    | Explain the Borel function theorem.  |         |     |
| 4      | 14.a.    | Explain the Relation between different modes of convergence.   |         |     |
|        |          | (OR)   |         | CO4 |
|        | 14.b.    | Explain the properties of convergence in r <sup>th</sup> mean.   |         |     |
|        | 15.a.    | Derive the Weak law of large numbers.  |         |     |
| 5      |          | (OR)   |         | CO5 |
|        | 15.b.    | Derive the Laplace distribution.   |         |     |

### SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks  $(3 \times 10 = 30)$ 

| Module<br>No. | Question No. | Question   | K Level | СО  |
|---------------|--------------|--|---------|-----|
| 1             | 16           | Derive the conditional expectation and conditional variance.                                 | K1      | CO1 |
| 2             | 17           | State and prove Lindeberg-Levy theorem.  | K2      | CO2 |
| 3             | 18           | State and prove Borel - cantelli Lemma (0-1 law).  | K1      | CO3 |
| 4             | 19           | Derive the Helly Bray lemma.   | K2      | CO4 |
| 5             | 20           | Define Central limit theorem and it's     Application     Define De-Moivre's Laplace theorem | K1      | CO5 |