18MCU22

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

REAL ANALYSIS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 Every infinite subset of a countable set A is _________.
(i) Countable (ii) not countable
(iii) Union (iv) Intersection

2 When a sequence $\{S_n\}$ of real number is said to be monotonically increasing?
(i) $S_n = S_{n+1}, (n = 1,2,3 \ldots)$ (ii) $S_n \leq S_{n+1}, (n = 1,2,3 \ldots)$
(iii) $S_n \geq S_{n+1}, (n = 1,2,3 \ldots)$ (iv) $S_n < S, (n = 1,2,3 \ldots)$

3 If f is continuous at every point of E, then f is said to be _____ on E.
(i) bounded (ii) unbounded
(iii) connected (iv) continuous

4 Let f be defined on $[a, b]$ and if f has a local maximum at a point $x \in (a, b)$ and if $f'(x)$ exists, then _______.
(i) $f'(x) \leq 0$ (ii) $f'(x) \geq 0$
(iii) $f'(x) = 0$ (iv) $f'(x) \neq 0$

5 If P^* is a refinement of P, then which one of the following is not true?
(i) $P \subset P^*$ (ii) $L(P^*, f, \alpha) \leq L(P, f, \alpha)$
(iii) $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ (iv) $L(P^*, f, \alpha) \leq U(P, f, \alpha)$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a) Let $\{E_n\}$, $n = 1,2,3 \ldots.$ be a sequence of countable sets and put $S = \bigcup_{n=1}^{\infty} E_n$. Then prove that S is countable.

OR

b) If E is an infinite subset of a compact set K, then show that E has a limit point in K.

7 a) Prove that a subset E of real line R^1 is connected if and only if it has the following property: If $x \in E, y \in E$ and $x < z < y$, then $z \in E$.

OR

b) If $\{P_n\}$ is a sequence in a compact metric space X, then prove that some subsequence of $\{P_n\}$ converges to a point of X.

8 a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

OR

b) Let f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that $f(X)$ is compact.

9 a) Let f is continuous on $[a, b]$, $f'(x)$ exists at some point $x \in [a, b]$, g is defined on an interval I which contains the range of f and g is differential at the point $f(x)$. If $h(t) = g(f(t))$ $a \leq t \leq b$, then h is differentiable at x and $h'(x) = g'\big(f(x)\big)f'(x)$. Justify the above statement.

OR

b) Let $f: [a, b] \to R^k$ be a continuous and let f be differentiable in (a, b). Then prove that there exists $x \in (a, b)$ such that $|f(b) - f(a)| \leq (b - a)|f'(x)|$.

Cont...

10 a) If f is continuous on $[a, b]$, then show that $f \in \mathcal{R}(\alpha)$ on $[a, b]$.

OR

b) State and Prove fundamental theorem of Calculus.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a) Prove that (i) Compact subsets of metric spaces are closed. (ii) Closed subsets of compact sets are compact.

OR

b) If X is a metric space and $E \subset X$, then show that
(i) $\bar{E}$ is closed
(ii) $E = \bar{E}$ if and only if E is closed.
(iii) $\bar{E} \subset F$ for every closed set $F \subset X$ such that $E \subset F$.

12 a) Prove that every k-cell is compact.

OR

b) If $\{s_n\}$, $\{t_n\}$ are complex sequences, and $\lim_{n\to\infty} s_n = s$, $\lim_{n\to\infty} t_n = t$, then show that
(i) $\lim_{n\to\infty}(s_n + t_n) = s + t$;
(ii) $\lim_{n\to\infty}(cs_n) = cs$, $\lim_{n\to\infty}(c + s_n) = c + s$ for any number c;
(iii) $\lim_{n\to\infty}(s_n t_n) = st$.

13 a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X.

OR

b) Let f be a continuous real function on the interval $[a, b]$. If $f(a) < f(b)$ and if c is a number such that $f(a) < c < f(b)$, then prove that there exists a point $x \in (a, b)$ such that $f(x) = c$.

14 a) State and Prove generalized Mean-value theorem.

OR

b) State and Prove Taylor's theorem.

15 a) Show that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.

OR

b) Assume α increases monotonically and $\alpha' \in \mathcal{R}$ on $[a, b]$. Let f be a bounded real valued function on $[a, b]$. Then prove that $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$ where $\int_a^b f\, d\alpha = \int_a^b f(x)\, \alpha'(x) dx$.

Z-Z-Z s END