22MCU101N/22MCU101

# PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

## BSc DEGREE EXAMINATION DECEMBER 2023
(First Semester)

Branch - **MATHEMATICS WITH COMPUTER APPLICATIONS**

## CALCULUS

Time: Three Hours

Maximum: 75 Marks

### SECTION-A (10 Marks)

Answer **ALL** questions

**ALL** questions carry **EQUAL** marks (10 × 1 = 10)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 1 | $\vec{a}X\vec{b}$ is orthogonal to ______<br>a) $\vec{a}$ b) $\vec{b}$<br>c) both $\vec{a}$ and $\vec{b}$ d) neither $\vec{a}$ nor $\vec{b}$ | K1 | CO1 |
| | 2 | If $\vec{r}(t) = \langle t^3, ln(3-t), \sqrt{t}\rangle$ then the domain of $\vec{r}$ is ___.<br>a) (0,3) b) [0.3]<br>c) (0,3] d) [0,3) | K2 | CO1 |
| 2 | 3 | If $f(x,y) = 4 - x^2 - 2y^2$, then $f_x(1,1) =$ _.<br>a) -4 b) -1<br>c) -2 d)-3 | K2 | CO2 |
| | 4 | The equation $\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ is called ___ equation<br>a) Laplace b) Wave<br>c) Euler d) Heat | K1 | CO2 |
| 3 | 5 | If $f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$ then $f(3,2) =$_____.<br>a) $\frac{\sqrt{6}}{2}$ b) $\frac{\sqrt{3}}{2}$<br>c) 0 d) 1 | K2 | CO3 |
| | 6 | If f(a,b) is neither a maximum nor minimum then (a,b) is a ____ point.<br>a) maximum b) minimum<br>c) saddle d) extreme | K1 | CO3 |
| 4 | 7 | If $f(x,y) \geq g(x,y)$ for all x in R then which of the following property holds:<br>a) $\iint_R f(x,y)\,dA \geq \iint_R g(x,y)\,dxdy$<br>b) $\iint_R f(x,y)\,dA \leq \iint_R g(x,y)\,dxdy$<br>c) $\iint_R f(x,y)\,dA = \iint_R g(x,y)\,dxdy$<br>d) $\iint_R f(x,y)\,dA = -\iint_R g(x,y)\,dxdy$ | K1 | CO4 |
| | 8 | If $R = [0,\pi/2]X[0,\pi/2]$ then $\iint_R \sin x \cos y\, dA =$_____<br>a) 0 b)1<br>c) -1 d) 2 | K2 | CO4 |
| 5 | 9 | If $B = \{(x,y) \mid 0 \leq x \leq 1, -1 \leq y \leq 2, 0 \leq z \leq 3\}$ then $\iiint_B xyz^2 dV =$____.<br>a) $\frac{27}{2}$ b) $-\frac{27}{2}$<br>c) $-\frac{27}{4}$ d) $\frac{27}{4}$ | K2 | CO5 |
| | 10 | The rectangular coordinates of the point with spherical coordinates $\left(2, \frac{\pi}{4}, \frac{\pi}{3}\right)$ is _____.<br>a) $\left(\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}, 1\right)$ b) $\left(-\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}, -1\right)$<br>c) $\left(\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}, -1\right)$ d) $\left(-\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}, 1\right)$ | K1 | CO5 |

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## SECTION - B (35 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 11.a. | Find the parametric equations and symmetric equations of the lines that passes through $A(2,4,-3)$ and $B(3,-1,1)$. At what points does this line intersect the $xy$ −plane? | K2 | CO1 |
| | | (OR) | | |
| | 11.b. | Find the curvature of the twisted cube $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ at a general point and at (0,0,0) | | |
| 2 | 12.a. | Examine whether $\lim_{(x,y)\to(0,0)} f(x,y)$ exists or not if $f(x,y) = \frac{xy}{x^2+y^2}$. | K3 | CO2 |
| | | (OR) | | |
| | 12.b. | Determine all the second derivatives of $f(x,y) = x^3 + x^2y^3 - 2y^2$ and examine whether $f_{xy} = f_{yx}$ or not. | | |
| 3 | 13.a. | a) If $f(x,y) = xe^y$, then find the rate of change of f at the point $P(2,0)$ in the direction from P to $Q(1/2, 2)$.<br>b) Illustrate the direction in which f have the maximum rate of change? What is the maximum rate of change? | K3 | CO3 |
| | | (OR) | | |
| | 13.b. | Make use of the concept of Lagrange multipliers to find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest and farthest from the point $(3,1,-1)$. | | |
| 4 | 14.a. | Evaluate $\iint_D (x + 2y)\, dA$ where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$. | K4 | CO4 |
| | | (OR) | | |
| | 14.b. | Evaluate $\iint_R (3x + 4y^2)\, dA$, where R is the region in the upper half plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. | | |
| 5 | 15.a. | Using triple integral, estimate to find the volume of the tetrahedron bounded by the planes $x + 2y + z = 2, x = 2y, x = 0, z = 0$. | K4 | CO5 |
| | | (OR) | | |
| | 15.b. | Evaluate $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV$ where B is the unit ball $B = \{(x,y,z) \mid x^2 + y^2 + z^2 \leq 1\}$. | | |

## SECTION -C (30 Marks)

Answer **ANY THREE** questions
ALL questions carry **EQUAL** Marks (3 × 10 = 30)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 16 | Identify the unit normal and the binormal vectors for the circular helix $\vec{r}(t) = \cos t\,\vec{i} + \sin t\,\vec{j} + t\vec{k}$. | K4 | CO1 |
| 2 | 17 | Show that $f(x,y) = xe^{xy}$ is differentiable at (1,0). Also identify its linearization and hence approximate $f(1.1, -0.1)$. | K4 | CO2 |
| 3 | 18 | Estimate the local maximum, minimum and saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$. | K4 | CO3 |
| 4 | 19 | The density at any point on a semi-circular lamina is proportional to the distance from the centre of the circle. Examine the centre of mass of the lamina. | K4 | CO4 |
| 5 | 20 | Analyze the formula for triple integration in spherical coordinates. | K4 | CO5 |

Z-Z-Z END