19MCU05

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(Second Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALYTICAL GEOMETRY OF 3D AND VECTOR CALCULUS

Time: Three Hours　　　　　　　　　　Maximum: 50 Marks

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5x1=5)

1. Find the common perpendicular vector of $\vec{a} = -\vec{\imath} + 3\vec{\jmath} + \vec{k}$ and $\vec{b} = \vec{\imath} + 2\vec{\jmath} + 2\vec{k}$
   a) $4\vec{\imath} - 2\vec{\jmath} + 2\vec{k}$　　b) $-4\vec{\imath} + 2\vec{\jmath} + 2\vec{k}$　　c) $4\vec{\imath} + +3\vec{\jmath} - 5\vec{k}$　　d) $-4\vec{\imath} + \vec{\jmath} + \vec{k}$

2. What is the equation of a sphere with centre (3, 2, -1) and radius 5?
   a) $(x - 3)^2 + (y - 2)^2 + (z + 1)^2 = 5^2$
   b) $(x + 3)^2 + (y - 2)^2 + (z - 1)^2 = 5^2$
   c) $(x - 3)^2 + (y + 2)^2 + (z - 1)^2 = 5^2$
   d) $(x + 3)^2 + (y + 2)^2 + (z - 1)^2 = 5^2$

3. Find the equation of the cone whose vertex is origin, axis is the line $x = y = z$ and semi vertical angle $cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
   a) $xy + yz + zx = 0$　　b) $xy + yz = 1$　　c) $yz + zx = 2$　　d) $yz + zx = 3$

4. The vector point function $\vec{F} = (2xy + z^3)\vec{\imath} + x^2\vec{\jmath} + 3xz^2\vec{k}$ is satisfies
   a) $\nabla \cdot \vec{F} = 0$　　b) $\nabla - \vec{F} = 0$　　c) $\vec{F} \times \nabla = 0$　　d) $\nabla \times \vec{F} = 0$

5. Stoke's theorem gives the relationship between
   a) Surface and volume integral　　b) Surface and line integral
   c) Line and volume integral　　d) Gradient and divergence

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5X3=15)

6. a) Find the equation of the plane which is perpendicular to $2\vec{\imath} + 3\vec{\jmath} + 5\vec{k}$ and passing through $(1,1,-1)$

(OR)

   b) The vertices of a tetrahedron are (0, 1, 2), (3, 0, 1), (4, 3, 6), (2, 3, 2); show that its volume is 6.

7. a) Find the centre and radii of the sphere. $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$

(OR)

   b) Find the equation of the circle which lies on the sphere $x^2 + y^2 + z^2 - 25 = 0$ and has the centre at $(1,2,3)$.

8. a) Find the equation of the cone whose vertex is the point $(1,1,0)$ and whose guiding curve is $y = 0, x^2 + z^2 = 4$

(OR)

   b) Find the enveloping cone of the sphere $x^2 + y^2 + z^2 - 2x + 4z = 1$ with its vertex at $(1,1,1)$.

Cont...

9. a) Find the value of b if $\nabla \cdot \vec{F} = 0$ where $\vec{F} = (z + 3y)\vec{i} + (x - 2z)\vec{j} + (x + bz)\vec{k}$

(OR)

b) Evaluate $\int_C y^2 dx + x\, dy$, where C is the line segment from $(-5, -3)$ to $(0,2)$

10. a) Evaluate $\oint_C (xy - x^2)dx + x^2y\, dy$ over the triangle bounded by the lines $y = 0, x = 1, y = x$

(OR)

b) Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ over the unit sphere $x^2 + y^2 + z^2 = 1$, where $\vec{F} = z\vec{i} + y\,\vec{j} + x\vec{k}$

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5X6=30)

11. a) Find the equation of the plane containing the line $\frac{1}{-3}(x + 1) = \frac{1}{2}(y - 3) = (z + 2)$ and the point $(0, 7, -7)$.

(OR)

b) Find the distance of the point $(3, -4, 5)$ from the plane $2x + 5y - 6z = 19$ measured along a line with direction cosines proportional to $(2,1,-2)$.

12. a) Find the equation of the sphere passing through the points $(4, -1,2)$, $(0, -2,3)$, $(1, -5, -1)$ and $(2,0,1)$

(OR)

b) Find the equations of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x - 4y = 0$, $x + 2y + 3z = 8$ and touch the plane $4x + 3y = 25$

13. a) Prove that the equation $4x^2 - y^2 + 2z^2 - 2xy - 3yz + 12x - 11y + 6z + 4 = 0$ represents a cone whose vertex is $(-1, -2, -3)$

(OR)

b) Find the equation of right circular cone with vertex $(2,3,1)$, axis parallel to the line $-x = \frac{y}{2} = z$ and one of its generators having direction cosines proportional to $(1, -1,1)$

14. a) Evaluate $\int_C (2 + x^2y)ds$, where C is the upper half of the unit circle $x^2 + y^2 = 1$

(OR)

b) Use Green's theorem to evaluate $\oint_C x^4\, dx + xy\, dy$ where C is the closed curve of the triangle of vertices $(0,0)$, $(1,0)$ and $(0,1)$

15. a) Use Stoke's theorem to evaluate $\iint_S \nabla \times \vec{F}.\vec{n}\, dS$ where S is the upper half of the hemisphere of radius a, centre at the origin and $\vec{F} = 2y\vec{i} - x\vec{j} + z\vec{k}$

(OR)

b) Verify Gauss's for $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ taken over the region bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$

Z-Z-Z

END