TOTAL PAGES: 2

**20MAP07**

**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**MSc DEGREE EXAMINATION MAY 2022**
(Second Semester)

Branch – **MATHEMATICS**

## PARTIAL DIFFERENTIAL EQUATIONS

Time: Three Hours | Maximum: 50 Marks

**SECTION-A (5 Marks)**
Answer **ALL** questions
**ALL** questions carry **EQUAL** marks (5 x 1 = 5)

1 The partial differential equation by eliminate the constants $a$ and $b$ from $z = (x+a)\ (y+b)$ is _______.
(i) $px + qy = q^2$ (ii) $z = pq$
(iii) $px - qy = 0$ (iv) $4xy = pq$

2 In a radio equation, $\frac{\partial^2\varphi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$ where $c =$_______.
(i) $Lc$ (ii) $(Lc)^{-1/2}$
(iii) $(Lc)^{1/2}$ (iv) $L/c$

3 $\nabla^2 + = -4\pi\rho$ is a _______ equation.
(i) Laplace's (ii) potential
(iii) Poisson's (iv) steady current

4 The one-dimensional wave equation is ________.
(i) $\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$ (ii) $\frac{\partial^2 y}{\partial x^2} = c^2\frac{\partial^2 y}{\partial t^2}$
(iii) $\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial \xi^2}$ (iv) $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$

5 The equation for the diffusion in Isotropic substances is ________.
(i) $\frac{\partial c}{\partial t} = D\nabla^2 C$ (ii) $\nabla^2\theta = \frac{\partial\theta}{\partial t}$
(iii) $\frac{\partial c}{\partial t} + div\, J = 0$ (iv) $\frac{\partial c}{\partial t} = div\,(D\ grad\ c)$

**SECTION - B (15 Marks)**
Answer **ALL** Questions
**ALL** Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Find the general solution of the differential equation .

$$x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z.$$

OR

b Find the complete integral of the equation $z^2 = pqxy$.

7 a Solve the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

OR

b Form a partial differential equation from.

$$z = f(x^2 - y) + g(x^2 + y).$$

8 a Write short notes on steady flow of heat.

OR

b Obtain the necessary condition for the existence of the solution of the interior Neumann problem.

**Cont...**

9 a Write short notes on electromagnetic waves.

OR

b The points of trisection of a string are pulled aside through a distance $\epsilon$ on opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the mid-point of the string always remains at rest.

10 a The faces $x = 0$, $x = a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slab is described by the equation $= f(x)$ $(0 \leq x \leq a)$. Determine the temperature at a subsequent time $t$.

OR

b Determine the Laplace inversion formula using Laplace transforms.

## SECTION -C (30 Marks)

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Find the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.

OR

b Show that the equations $xp - yq = x$, $x^2p + q = xz$ are compatible and find their solution.

12 a Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ into canonical form.

OR

b Determine the solution of the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^2 z}{\partial y^2} = 0$,

$(-\infty < x < \infty, y \geq 0)$ satisfying the conditions:

(i) $z$ and its partial derivatives tend to zero as $x \to \pm\infty$.

(ii) $z = f(x)$, $\frac{\partial x}{\partial y} = 0$ on $y = 0$.

13 a Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can form a family of equipotential surfaces, and find the general form of the corresponding potential function.

OR

b Derive the solution of Laplace equation by using method of separation of variables in polar coordinates.

14 a A thin membrane of great extent is released from rest in the position $z = f(x, y)$. Determine the displacement at any subsequent time.

OR

b Derive the D'Alembert's solution of the one-dimensional wave equation.

15 a State and prove Duhamel's theorem

OR

b Find the solution of the equation $k\nabla^2\theta = \frac{\partial \theta}{\partial t}$ for an infinite solid whose initial distribution of temperature is given by $\theta(r, 0) = f(r)$ where the function $f$ is prescribed.

Z-Z-Z

END