20MAP09

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2022
(Second Semester)

Branch – **MATHEMATICS**

MATHEMATICAL STATISTICS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The expected value of the random variable X if $x_1 = -1$; $p_1 = 0.1$ and $x_2 = 1$; $p_2 = 0.9$ is________
   i. 0.9
   ii. 0.8
   iii. 0.7
   iv. 0.6

2. Binomial distribution P(X=k) is ________
   i. $\binom{n}{k} p^k (1-p)^{n-k}$
   ii. $p^k (1-p)^{n-k}$
   iii. $\binom{n}{k} (1-p)^{n-k}$
   iv. $\binom{n}{k} p^k$

3. The sequence $\{X_n\}$ of a random variable is called stochastically convergent to zero if it satisfies ________.
   i. $\lim_{n\to\infty} P(|X_n| < \epsilon) = 0$
   ii. $\lim_{n\to\infty} P(|X_n| > \epsilon) = 0$
   iii. $\lim_{n\to\infty} P(|X_n| \geq \epsilon) = 0$
   iv. $\lim_{n\to\infty} P(|X_n| \leq \epsilon) = 0$

4. Chapman- Kolmogorov equation is ________.
   i. $p_{ij}(t_1, t_2) = \sum_k p_{ik}(t_1, s) p_{kj}(s, t_2)$
   ii. $p_{ij}(t_1, t_2) = \sum_k p_{ik}(t_1, s)$
   iii. $p_{ij}(t_1, t_2) = \sum_k p_{ik}(t_1, s)$
   iv. $p_{ij}(t_1, t_2) = \sum_k p_{ik}(t_1, t_2)$

5. A random sample is called ____________ if the random variables $X_1, X_2, X_3, \ldots$ are independent.
   i. Sample space
   ii. Random sample
   iii. Simple
   iv. Sample mean

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Show that if g_1 and g_2 are single valued functions then.

$$E(g_1(x) + g_2(x)) = E(g_1(x)) + E(g_2(x))$$

OR

b Find the characteristic function and moments of the normal distribution $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$.

7 a State and prove the addition theorem for the binomial distribution.

OR

b The random variable X has the distribution N(1;2). Find the probability that X is greater than 3 in absolute value.

Cont...

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Cont...

8 a State and prove Bernouli's Law of large numbers.

OR

b We throw a coin n=100 times. We assign the number 1 to the appearance of heads and the number 0 to the appearance of tails. The probability of each of these events is equal to p=q=0.5. What is the probability that heads will appear more than 50 times and less than 60 times using de Moivre theorem.

9 a State the conditions satisfied by the Poisson process.

OR

b A function R(τ) is the correlation function of a process $\{Z_t, -\infty < t < \infty\}$ stationary in the wide sense continuous and satisfying m=0, $\sigma = 1$ if and only if there exists a distribution function F(λ) such that R(τ) = $\int_{-\infty}^{\infty} e^{i\lambda r}\ F(\lambda)$.

10 a Write short notes on arithmetic mean of independent normally distributed random variable.

OR

b Prove that the sequence $\{F_n(t)\}$ of distribution functions of students t with n degree of freedom satisfies for every t the relation $\lim_{n\to\infty}(F_n(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t} e^{-\frac{t^2}{2}}\,dt$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a State and prove Chebyshev inequality.

OR

b Find the density of the random variable X whose characteristic function is.

$$\varphi_1(t) = \begin{cases} 1-|t| & for\ |t| \le 1 \\ 0 & for\ |t| > 1 \end{cases}$$

12 a Fit a Poisson distribution for the following data.

X	:	0	1	2	3
Frequency	:	0.545	0.325	0.110	0.015
Probability	:	0.544	0.331	0.101	0.021

OR

b Give a brief note on Gamma distribution.

13 a State and prove Levy-Cramer theorem.

OR

b State and prove Lenderberg-Levy theorem.

14 a A stochastic process $\{X_t, 0 \le t \le \infty\}$ where X_t is the number of signals in the interval [0, t) satisfying conditions I to III and the equality P($X_0 = 0$) = 1 is a homogenous Poisson process.

OR

b The solutions $V_m(t)$of the system V_0 *and* V'_m with the initial conditions $V_m(0) = 1$ for $m = 1$ *and* 0 *for* $m \ne 0$ satisfy the relation $\sum V_m(t) = 1$ if and only if $\sum_{m=0}^{\infty} \frac{1}{\lambda_{i+m}} = \infty$.

15 a The random variables X_k $(k = 1,2,3..8)$ are independent and have the same normal distribution N(0;2). We consider the statistics $\psi^2 = \sum_{k=1}^{8} X_k^2$.

OR

b Find the distribution of the statistics U= S^2.

Z-Z-Z END