20MAP21

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2022
(Fourth Semester)

Branch – **MATHEMATICS**

MATHEMATICAL METHODS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. An integral equation is an equation in which an unknown function appears under __________ integral sign.
   (i) one (ii) two
   (iii) more (iv) one or more

2. In the fredholm integral equation of the first kind h(s) = ___.
   (i) 0 (ii) finite
   (iii) infinite (iv) < 0

3. $\Gamma(s,t;\lambda) = \sum_{m=1}^{\infty} \lambda^{m-1} k_m(s,t)$ is _____________ for all values of s, t in $|\lambda| < {}^{1}/_{B}$.
   (i) uniformly convergent (ii) divergent
   (iii) absolutely convergent (iv) both (i) and (iii)

4. ODE of first order can be solved by ___________ method.
   (i) Picard (ii) Jacobian
   (iii) Taylor (iv) Volterra

5. The boundary value problem in ODE lead to __________.
   (i) Volterra-type integral equations (ii) Fredholm-type integral equation
   (iii) both (i) & (ii) (iv) Neither (i) and (ii)

6. An integral equation is called singular if the range of integration is _____________.
   (i) Indefinite (ii) finite
   (iii) infinite (iv) Indeterminant

7. Functionals are ____________ quantities.
   (i) constant (ii) variable
   (iii) maximal (iv) minimal

8. A necessary condition for the extremum of φ(α) for α = 0 is $\varphi'(0) =$ ______.
   (i) < 0 (ii) > 0
   (iii) = 0 (iv) ∞

9. If on a plane one and only one curve of a family of curves passes through every point of a certain region, then the family of curves is said to form a __________.
   (i) pencil (ii) closed curve
   (iii) region (iv) field

10. The condition of possibility of constructing a field of extremals including a given extremal is called _______ condition.
    (i) Euler (ii) Jacobi
    (iii) Hamilton (iv) Abel's

Cont...

SECTION - B (35 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 7 = 35)

11 a State and prove Fredholm theorem.

OR

b Solve $g(s) = s + \lambda \int_0^1 (st^2 + s^2 t) g(t) dt$.

12 a Solve $g(s) = s + \lambda \int_0^1 e^{s-t} g(t) dt$.

OR

b Solve $g(s) = s + \lambda \int_0^1 (st + (st)^{1/2}) g(t) dt$.

13 a Reduce the BVP $y''(s) + \lambda P(s) y = Q(s)$ with y(a) = 0, y(b) = 0 to a Fredholm integral equation.

OR

b Solve $f(s) = \int_a^s \frac{g(t)}{(\cos t - \cos s)^{1/2}}, 0 \le a < s < b \le \pi$.

14 a State and prove fundamental lemma of calculus of variation.

OR

b On what curves can $V[y(x)] = \int_0^1 [(y')^2 + 12xy]\, dx, y(0) = 0, y(1) = 1,$ be extremized?

15 a Is the Jacobi condition fulfilled for the extremal of $V = \int_0^a (y'^2 - y^2) dx$ that passes through A(0,0) and B(a,0)?

OR

b Test for an extremum of $V[y(x)] = \int_0^a (y')^3\, dx, y(0) = 0, y(a) = b, a > 0,$ $b > 0$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Find the eigenvalues and eigen functions of $g(s) = \lambda \int_1^2 \left[st + \frac{1}{st}\right] g(t) dt$.

17 Solve: $g(s) = 1 + \lambda \int_0^{\pi} [\sin(s + t)] g(t) dt$.

18 Reduce $y''(s) + A(s) y' + B(s) y = F(s), y(a) = y_0, y(b) = y_1$ to Fredholm integral equation.

19 State minimum surface of revolution problem and solve it.

20 Find the equation of geodesics on a surface on which the element of length of the curve is of the form $ds^2 = [\varphi_1(x) + \varphi_2(y)](dx^2 + dy^2)$.

Z-Z-Z END