23DAU315

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(Third Semester)

Branch - **COMPUTER SCIENCE WITH DATA ANALYTICS**

LINEAR ALGEBRA

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. A unit vector u is a vector whose length equals -------.
   (i) 1 (ii) 0 (iii)-1 (iv) 2

2 The system $Ax = b$ is ------- if and only if b is in the column space of A.
   (i) bounded (ii) solvable
   (iii) unsolvable (iv) unbounded

3 Any n vectors that span $\mathbb{R}^n$ must be -----------.
   (i) dependent (ii) independent
   (iii) null space (iv) subspace

4 The determinant is zero when the matrix has ------.
   (i) inverse (ii) no inverse
   (iii) singular (iv) non singular

5 The transformation is linear only if -----.
   (i) $v_0 = 1$ (ii) $v_0 = 0$ (iii) $v_0 = -1$ (iv) $v_0 = c$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Find a unit vector u in the direction of $v = (3,4)$. Find a unit vector U that is perpendicular to u. How many possibilities for U?

OR

b Find $cos\,\theta$ for $v = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $w = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

7 a Describe the nullspace of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$.

OR

b Write the properties of a matrix with full row rank.

8 a Find the projection matrix $P = \frac{aa^T}{a^Ta}$ onto the line through $a = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$.

OR

b The equation $x - 3y - 4z = 0$ describes the plane P in R^3

(i) The plane P is the nullspace $N(A)$ of what 1×3 matrix A?

9 a Compute the determinants of the matrix by row operations: $A = \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{pmatrix}$.

OR

b Find the cross product of $u = (3,2,0), v = (1,4,0)$ in the XY plane.

10 a T rotates every vector by the angle θ. Here $V + W = R^2$. Find A.

OR

b The space of 2×2 matrices has the following four vectors as a basis:

$$v_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, v_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, v_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

T is the linear transformation that transpose every 2×2 matrix. What is the matrix that represents T in this basis? What is the inverse matrix A^{-1}?

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Describe the column picture of the following system of three equations Ax=b. Solve by careful inspection of the columns :

$x + 3y + 2z = -3; 2x + 2y + 2z = -2; 3x + 5y + 6z = -5.$

OR

b Multiply the following matrices in the orders EF and FE:

$E = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{pmatrix}, F = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{pmatrix}$. Also compute $E^2 = EE, F^3 = FFF$ and produce F^{100}.

12 a Let $A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$ then find the reduced echelon form of A. What is the rank? What is the special solution to Ax=0?.

OR

b Find the rank of the matrix $A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$.

13 a If $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ find $\hat{x}$ and p and P.

OR

b Project the vector $b = (2,3,4)$ onto the line through $a = (2,2,1)$ and then onto the plane that also contains $a^* = (1,0,0)$. Check that the first error vector $b - p$ is perpendicular to a, and the second error vector $e^* = b - p^*$ is also perpendicular to a^*. Find the 3×3 projection matrix P onto that plane of a and a^*. Find a vector whose projection onto the plane is the zero vector.

14 a Find the eigenvalues and eigenvectors of A and A^2 and A^{-1} and $A + 4I$: where $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$.

OR

b Ax=b is $\begin{pmatrix} 2 & 6 & 2 \\ 1 & 4 & 2 \\ 5 & 9 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ Use Cramer's rule with ratios $\frac{\det B_j}{\det A}$ to solve Ax=b. Also find the inverse matrix $A^{-1} = C^{-1}/\det A$. Why is the solution x for this b the same as column 3 of A^{-1}? Which cofactors are involved in computing that column x?

15 a S rotates by θ and T rotates by $-\theta$. Then prove $TS = I$ matches $AB = I$.

OR

b Find the eigenvalues and eigenvectors of the matrix $P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

Z-Z-Z END