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22STP313

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2024
(Third Semester)

Branch – **STATISTICS**

STOCHASTIC PROCESSES

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 If $\{X(t), t \in T\}$ is a stochastic process in which X(t) represents the amount of rainfall on the t-th day of a month, the stochastic process is called
(i) Discrete Random Sequence (ii) Discrete random Process
(iii) Continuous Random Sequence (iv) Continuous random Process

2 A state i is aperiodic if its period is equal to
(i) 0 (ii) 1
(iii) infinity (iv) even number

3 The relative importance of websites is determined by search engines using the following concept.
(i) Decomposition (ii) Variable reduction
(iii) Random walk (iv) Recurrence

4 The Poisson process satisfies
(i) Additive property (ii) Markovian property
(iii) either (i) or (ii) (iv) Both (i) and (ii)

5 Which of the following is true about $E[X(t1)X(t2)]$ in a wide sense stationary process?
(i) is always 0 (ii) Depends only on time difference
(iii) Depends only on t_1 (iv) Depends only on t_2

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Explain in the classification of Stochastic Processes.

OR

b Explain the concept of irreducible Markov chain by describing an equivalence class.

7 a Explain the process of identifying a recurrent state using the probability of ultimate return by giving the relevant expressions.

OR

b Prove that the Chapman Kolmogorov equation gives the higher order transition probabilities.

8 a Explain the gamblers ruin process and derive the probability that the first player is ultimately ruined.

OR

b Derive the Kolmogorov backward equation.

Cont...

9 a Prove that Poisson process is a Markov Process.

OR

b Prove that the Poisson process has additive property.

10 a Verify whether the Stochastic process $\{X(t), t \in T\}$, is covariance stationary where X(t) = A cos t+ B sin t . Here A and B are independent random variables with 0 means and equal variances.

OR

b Prove that $\{X(t), t \in T\} where\ X(t) = \theta\varepsilon_{t-1} + \varepsilon_t$ is stationary.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Explain the terms Recurrence, Null recurrent state and aperiodic states.

OR

b Verify whether the Markov chain with states{A,B,C} is irreducible if the TPM

is $P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

12 a Identify the aperiodic states of the Markov chain if the TPM is

$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}$

OR

b Find the stationary distribution of a Markov chain whose Transition Probability matrix is $P = \begin{pmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \end{pmatrix}$

13 a Derive an expression for the probability of never being absorbed in to a recurrent class, starting from a state i.

OR

b In a gambler's ruin problem on n+1 states, derive an expression for the probability that the process ultimately enters in to an absorbing state 0.

14 a Derive the differential difference equation of Birth Death process.

OR

b Derive the Moment Generating function of a Poisson process.

15 a Prove that for a renewal process $M(t) = F(t) + \int_0^t M(t-x)f(x)dx$

OR

b Prove that $M(t) = \sum_{n=1}^{\infty} F_n(t)$ in a renewal process.

Z-Z-Z END