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22STP102N/22STP102

PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2024

(First Semester)

Branch - **STATISTICS**

ADVANCED PROBABILITY THEORY

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If C is constant then $E(cX) =$ _______. a) $E(Xc)$ b) $cE(X)$ c) $C^2E(X)$ d) $c + E(X)$	K1	CO1
	2	$E\|XY\| \leq \sqrt{E\|X\|^2 E\|Y\|^2}$ is called ___inequality. a)Schwartz b) Holder's c) C_r d) Murkowski	K2	CO1
2	3	If ϕ is the characteristic function of a general distribution function of F, then ϕ is ________. a) Complex b) continuous c) unique d) converges	K1	CO2
	4	If $F_n \xrightarrow{w} F$, then F is ____________. a) unique b) countable c) converges c) measurable	K2	CO2
3	5	Two events A and B are said to be independent of $P(A \cap B) =$ ____________. a) $A\,P(B)$ b) $P(B)B$ c) $P(A) + P(B)$ d) $P(A)P(B)$	K1	CO3
	6	Borel function of independent random variables are______. a) dependent b) independent c) zero d) one	K2	CO3
4	7	$X_n \xrightarrow{p} X$ and $X_n \xrightarrow{p} X'$ $\Rightarrow X$ and X' are ___. a) equivalent b) converges c) zero d) different	K1	CO4
	8	$X_n \xrightarrow{L} X$ and $X_n \xrightarrow{L} C$, then $X_nY_n \xrightarrow{L}$ _______. a) $X + C$ b) X/C c) CX d) $C?X$	K2	CO4
5	9	The sequence of independent random variables WLLN hold if $\sum_1^n \frac{\sigma_k^2}{n^2} = V\left(\frac{S_n}{n^2}\right) \to$ ________. a) 0 b) ∞ c) $-\infty$ d) 1	K1	CO5
	10	If $\sum_1^n X_k = S_n \to S < \infty$ and $b_n \uparrow \infty$ then $\left(\frac{1}{b_n}\right)\sum_1^n b_n x_k \to$ _____. a) ∞ b) $-\infty$ c) 0 d) 1	K2	CO5

SECTION – B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	State and prove holder's inequality.	K2	CO1
		(OR)		
	11.b.	If $x > 0$ and is integrable, X can be infinite at most on a set of probability measure zero.		

Cont...

2	12.a.	Let $X_n \xrightarrow{L} X$ and for some $p > 0$ $\underset{n}{Sup}\, E\lvert X_n^p\rvert = M < \infty$. Then for all $r < p, E\lvert X_n^r\rvert \to E\lvert X_n\rvert^r < \infty$	K4	CO2
		(OR)		
	12.b.	State and prove second limit theorem.		
3	13.a.	If $X_n's$ are independent and $X_n \to 0\ (a.s)$, then show that $\sum P[\lvert X_n\rvert \geq c] < \infty$ whatever be $c > 0$, finite.	K3	CO3
		(OR)		
	13.b.	Prove that sub classes of independent classes are independent.		
4	14.a.	$X_n \xrightarrow{p} 0$ iff $E\left(\frac{\lvert X_n\rvert}{(1+\lvert X_n\rvert)}\right) \to 0$ as $n \to \infty$.	K5	CO4
		(OR)		
	14.b.	Let $X_n \xrightarrow{p} X$ and $Y_n \xrightarrow{p} Y$. Then prove that a)a $X_n \xrightarrow{p} aX\ (a, real)$ b) $X_n + Y_n \xrightarrow{p} X + Y$		
5	15.a.	If $X_n's$ are uniformly bounded and $\sum X_n$ converges $a.s$ then $\sum \sigma_n^2$ and $\sum EX_n$ converge.	K5	CO5
		(OR)		
	15.b.	State and prove that Lindeberg –Levy theorem.		

SECTION –C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove C_r inequality.	K3	CO1
2	17	State and prove Bochner's theorem.	K4	CO2
3	18	State and prove Borel 0-1 law.	K4	CO3
4	19	Prove that $X_n \xrightarrow{p} X \Rightarrow F_n(X) \to F(x), x \in C(f)$	K5	CO4
5	20	If $\sum \sigma^2 < \infty$ then $\sum_n X_n - E(X_n))$ converges $a.s$ of $X_n's$ are $a.s$ bounded, converges is also true and we have $\sum \sigma^2 < \infty \Leftrightarrow \sum(X_n - EX_n)$ converges.	K5	CO5

Z-Z-Z END