

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc(SS) DEGREE EXAMINATION MAY 2024
(Second Semester)

Branch – SOFTWARE SYSTEMS (five year integrated)

DISCRETE STRUCTURES AND APPLIED GRAPH THEORY

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(5 x 1 = 5)

- What rule of inference is used here?
"It is cloudy and drizzling now. Therefore, it is cloudy now."
i) Addition
ii) Simplification
iii) Resolution
iv) Conjunction
- Let a set $S = \{2, 4, 8, 16, 32\}$ and \leq be the partial order defined by $S \leq R$ if a divides R .
Number of edges in the Hasse diagram of is _____.
i) 6
ii) 5
iii) 9
iv) 4
- Which of the following statements for a simple graph is correct?
i) Every path is a trail
ii) Every trail is a path
iii) Every trail is a path as well as every path is a trail
iv) Path and trail have no relation
- The total number of edges present in Complete graph K_n are _____.
i) $n(n-1)$
ii) $\frac{n-1}{2}$
iii) $\frac{n(n-1)}{2}$
iv) $\frac{n(n+1)}{2}$
- The sequence $\langle 000, 100, 110, 010, 011, 111, 101, 001 \rangle$ is a gray code of order _____.
i) 3
ii) 2
iii) 4
iv) none of the above

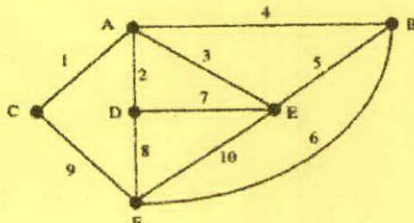
SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks

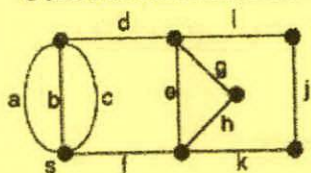
(5 x 3 = 15)

- Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent without using truth table.
(OR)
 - Prove that $\sqrt{2}$ is irrational.
- Using Warshall Algorithm to find the transitive closure of the relations $\{(1,2), (2,1), (2,3), (3,4), (4,1)\}$ on the set $\{1,2,3,4\}$
(OR)
 - Show that the relation R on a set A is Transitive if and only if $R^n \subseteq R$, for $n=1,2,3,\dots$
- Prove that the sum of the degrees of the vertices of a graph is twice the number of edges.
(OR)
 - From the given graph draw the following:
(i) Walk of length 6
(ii) Is this an Euler Graph? Give reason.
(iii) Is this an Hamiltonian path for this Graph? Give reason.



Cont...

9. a) Draw Harary graph $H_{5,8}$.
(OR)
b) Prove that the Harary graph is k -Connected.
10. a) Construct an Eulerian tour of the given graph.



- (OR)
b) Explain Nearest Neighbor algorithm with example.

SECTION - C (30 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks

(5 x 6 = 30)

11. a) Show that the premises "A student in this class has not read the book", and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book".
(OR)
b) i) If $H_n = 1 + \frac{1}{2} + \dots + \frac{1}{n}$, $n \geq 1$, Using induction show that $H_{2^n} \geq 1 + \frac{n}{2}$.
12. a) Let $A = \{1, 2, 3, 4, 5, 6\}$. Define aRb if $a-b$ is divisible by 2 and aSb if $a \leq b$. Find the relation matrices of R, S and also find $M_{R \circ S}$, $M_{S \circ R}$, $M_{R \cup S}$.
(OR)
b) Let R be an equivalence relation on a set A . The following Statements are equivalent:
i) aRb ii) $[a] = [b]$ iii) $[a] \cap [b] \neq \Phi$
13. a) A graph G is bipartite if and only if it has no cycles of odd length.
(OR)
b) i) Prove that $P = \langle 4, 4, 4, 2, 2, 2 \rangle$ is graphical.
ii) Draw Complete bipartite graph $K_{4,3}$.
iii) Define Eccentricity, Diameter and Radius with example.
iv) Define Hamiltonian graph with example.
14. a) Let e be any edge of a k -connected graph G , for $k \geq 3$. Then prove that the edge-deletion subgraph $G-e$ is $(k-1)$ -connected.
(OR)
b) State and Prove Whitney-Synthesis Theorem.
15. a) A connected graph G has an open Eulerian trail if and only if it has exactly two vertices of odd degree.
(OR)
b) Let G be a simple n -vertex graph, where $n \geq 3$ such that $\deg(x) + \deg(y) \geq n$ for each pair of non-adjacent vertices x & y , then G is Hamiltonian.