20SSP03

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc(SS) DEGREE EXAMINATION MAY 2024
(First Semester)

Branch – **SOFTWARE SYSTEMS (five year integrated)**

CALCULUS AND ITS APPLICATIONS

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 1 = 10)

1 Find the integer floor function value of $\lfloor -1.2 \rfloor$.
(i) -2 (ii) -1 (iii) 1 (iv) 2

2 Identify $\frac{dy}{dx}$ using fundamental theorem if $y = \int_a^x (t^3 + 1)dt$.
(i) $\frac{x^4}{4} - \frac{a^4}{4}$ (ii) $x^3 - 1$ (iii) $\frac{x^4}{4}$ (iv) $x^3 + 1$

3 When the series $\lim_{n\to\infty} \left(\frac{a_{n+1}}{a_n}\right) = \rho$ converges if and only if
(i) ρ>1 (ii) ρ<1
(iii) ρ = 1 (iv) ρ ≠ 1

4 Name the test for the series $\sum_{n=1}^{\infty} \frac{2n+1}{n^2+2n+1}$ to prove diverges.
(i) Absolute Convergence Test (ii) The Ratio Test
(iii) Limit Comparison Test (iv) The Root Test

5 Find the value of $\frac{\partial f}{\partial x}$ at the point (4,-5) if $f(x, y) = x^2 + 3xy + y - 1$.
(i) 7 (ii) 6 (iii) -1 (iv) -7

6 Identify the interior point of the function $f(x, y) = 2 + 2x + 2y - x^2 - y^2$.
(i) (0,1) (ii) (1,1) (iii) (1,0) (iv) (2,2)

7 Which equation plays an important role in Population dynamics?
(i) Logistic equation (ii) Dynamic equation
(iii) Homogeneous equation (iv) Continuity equation

8 Choose the following equation which satisfies linearly independent on
$y_1 = x^2, y_2 = 5x, y_3 = 2x$
(i) $y_1 = y_2 + 2y_3$ (ii) $y_2 = 0y_1 + 2.5y_3$
(iii) $y_3 = 0y_1 + 2y_2$ (iv) $y_1 = y_2 + y_3$

9 Find a_0 in the Fourier Series.
(i) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$ (ii) $\frac{1}{2\pi}\int_0^{\pi} f(x)dx$
(iii) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$ (iv) $\frac{1}{\pi}\int_0^{2\pi} f(x)dx$

10 Identify RLC Circuit governed ODE equations.
(i) $LI'' + RI' + \frac{1}{C}I = E'(t)$ (ii) $LI'' + I' - \frac{1}{C}I = E'(t)$
(iii) $LI'' + RI' + CI = E'(t)$ (iv) $LI'' + RI' - \frac{1}{C}I = E(t)$

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks   (5 x 5 = 25)

11 a Sketch the function $y = x^2$ over the interval $[-2,2]$.
OR
b Determine the value of $\lim_{n\to 0}\frac{\sin 2x}{5x}$ and also examine $\lim_{t\to 0}\frac{\tan t \sec 2t}{3t}$.

Cont...

12 a Analyze the sequence whose n[th] term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ converges? And also find $\lim_{n\to\infty} a_n$.

OR

b Evaluate the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$ and also find if any where does the series converges to $\frac{1}{x}$.

13 a Determine $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at (0,0,0) if $x^3 + z^2 + ye^{xz} + z\cos y = 0$.

OR

b Sketch and find the tangent plane and normal line of the surface $f(x,y,z) = x^2 + y^2 + z - 9 = 0$ at the point (1,2,4).

14 a Solve $2xyy' = y^2 - x^2$ using extended method of reduction to separable form.

OR

b Evaluate the fourth order ODE $y^{iv} - 5y'' + 4y = 0$.

15 a Show the Fourier series of the function $f(x) = \begin{cases} -k & if -2 < x < 0 \\ k & if \quad 0 < x < 2 \end{cases}$ $p = 2L = 4, L = 2$.

OR

b Analyze the minimum square error E of F(x) with $N = 1,2 \ldots.10,20, \ldots..100$ and 1000 relative to $f(x) = x + \pi(-\pi < x < \pi)$ on the interval $-\pi \le x \le \pi$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Evaluate (i) $\lim_{x\to 1} \frac{x^2+x-2}{x^2-x}$ (ii) $\lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2}$.

OR

b Interpret the scenario of a heavy rock blown straight up from the ground by a dynamite blast. The Velocity of the rock at any time t during its motion was given as $v(t) = 160 - 32t$ ft/sec. Find the displacement of the rock during the time period $0 \le t \le 8$ and also calculate the total distance travelled during this time period.

17 a Assess the convergence of the following series (i) $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$ (ii) $\sum_{n=1}^{\infty} \frac{4^n n!\,n!}{(2n)!}$.

OR

b Determine the first few terms of the Taylor series for the given function using power series operations (i) $\frac{1}{3}(2x + x\cos x)$ (ii) $e^x \cos x$.

18 a Evaluate $\lim_{(x,y)\to(0,0)} \frac{4xy^2}{x^2+y^2}$ if it exists.

OR

b Find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

19 a Solve $\cos(x+y)\,dx + (3y^2 + 2y + \cos(x+y))\,dy = 0$.

OR

b Formulate the Fourier Cosine and Sine integrals of $f(x) = e^{-kx}$ where $x > 0$ and $k > 0$.

20 a Determine the two-half range expansion of the function

$$f(x) = \begin{cases} \frac{2k}{L}x & if\ 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & if\ \frac{L}{2} < x < L \end{cases}.$$

OR

b Develop the Fourier series for $f(x) = x^2$ in $-\pi < x < \pi$. Hence show that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$ using Parswal's Theorem.

Z-Z-Z END