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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc(SS) DEGREE EXAMINATION MAY 2024
(Fourth Semester)

Branch – **SOFTWARE SYSTEMS (five year integrated)**

APPLIED LINEAR ALGEBRA

Time: Three Hours — Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The homogenous equation $AX = 0$ has a _____ solution if and only if the equation has at least one free variable.
   (i) trivial (ii) non-trivial
   (iii) finite (iv) infinite

2. The linear combination of $(1,2,-1)$ *and* $(16,4,2)$ is ____.
   (i) $(9,2,7)$ (ii) $(4,-1,8)$
   (iii) both (i) & (ii) (iv) none of these

3. If A is orthogonal, then $|A| =$ ___.
   (i) 0 (ii) ± 1
   (iii) ± 2 (iv) $\pm\infty$

4. Two similar matrices have same ____.
   (i) determinant (ii) rank
   (iii) nullity (iv) all the above

5. The nature of the quadratic form $Q(x) = 3x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 + 4x_2x_3$ is ____.
   (i) positive definite (ii) positive semi-definite
   (iii) indefinite (iv) negative definite

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) List the general solution of the linear system whose augmented matrix has been reduced to
$$\begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$
(OR)
   b) Let $a_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, a_2 = \begin{bmatrix} 5 \\ -13 \\ -3 \end{bmatrix}$ and $b = \begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}$. Then span$\{a_1, a_2\}$ is a plane through the origin in $\mathbb{R}^3$. Is b in the plane? -Examine.

7. a) Investigate whether $v_1(1,1,2), v_2 = (1,0,1), v_3 = (2,1,3)$ span the vector space $\mathbb{R}^3$.
(OR)
   b) Deduce that $f_1 = 1, f_2 = e^x, f_3 = e^{2x}$ form a linearly independent set of vectors in $C^2[-\infty, \infty]$.

8. a) Let $\mathbb{R}^4$ have the Euclidean inner product. Calculate the cosine of the angle θ between the vectors $u = (4,3,1,-2), v = (-2,1,2,3)$.
(OR)
   b) Sketch the unit circle in an xy-coordinate system in $\mathbb{R}^2$ using the weighted Euclidean inner product $\langle u, v\rangle = \frac{1}{9}u_1v_1 + \frac{1}{4}u_2v_2$.

9. a) Consider the basis $S = \{v_1, v_2, v_3\}$ for $\mathbb{R}^3$, where $v_1 = (1,1,1), v_2 = (1,1,0), v_3 = (1,0,0)$. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$, be the linear transformation such that $T(v_1) = (1,0)$, $T(v_2) = (2,-1), T(v_3) = (4,3)$. Find a formula for $T(x_1, x_2, x_3)$ and hence compute $T(2,-3,5)$.
(OR)
   b) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix}$. Find $det(T)$ with respect to the basis $B' = \{u_1, u_2\}$ where $u_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Cont...

10. a) Estimate the eigenvalues and their multiplicities of $A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$:

(OR)

b) Let $A = \begin{bmatrix} 7 & 2 \\ -6 & 1 \end{bmatrix}$. Derive a formula for A^k given $A = PDP^{-1}$ where $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Identify all the solutions of $AX = b$ where $A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix}$ and $b = \begin{bmatrix} 7 \\ -1 \\ -4 \end{bmatrix}$.

(OR)

b) Find LU decomposition of the coefficient matrix $\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

12. a) The set $S = \{v_1, v_2, v_3\}$ where $v_1 = (1,2,1), v_2 = (2,9,0), v_3 = (3,3,4)$ is a basis for $\mathbb{R}^3$.- Justify.

(OR)

b) Determine the rank, nullity and hence the dimension of the matrix

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

13. a) Consider the vector space $\mathbb{R}^3$ with the Euclidean inner product. Apply the Gram Schmidt process to transform the basis vectors $u_1 = (1,1,1), u_2 = (0,1,1), u_3 = (0,0,1)$ into an orthogonal basis $\{v_1, v_2, v_3\}$. Then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{q_1, q_2, q_3\}$.

(OR)

b) Find the QR decomposition of $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$.

14. a) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$.

Determine the matrix for the transformation T with respect to the (i) standard basis and (ii) basis $B = \{u_1, u_2\}$ for $\mathbb{R}^2$ and $B'\{v_1, v_2, v_3\}$ for $\mathbb{R}^3$ where $u_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$.

(OR)

b) Evaluate the eigenvalues and basis for the Eigen spaces of the linear operator $T: P_2 \to P_2$ defined by $T(a + bx + cx^2) = -2c + (a + 2b + c)x + (a + 3c)x^2$.

15. a) Orthogonally diagonalize the matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ whose characeriscstic equation is $(\lambda - 7)^2(\lambda + 2) = 0$.

(OR)

b) Compute the singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.

Z-Z-Z

END