22MAP208N/ 22MAP208

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2024
(Second Semester)

Branch – **MATHEMATICS**

TOPOLOGY

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If X is any set the collection of all subsets of X is a Topology on X it is called ……. Topology (a) Discrete (b) Indiscrete (c) Trivial (d) Standard	K1	CO1
	2	Let Y be a subspace of X. If U is open in Y and Y is open in X then U is (a) Closed in X (b) Bounded in X (c) Open in Y (d) Open in X	K2	CO1
2	3	Let X and Y be topological spaces. Let $f: X \to Y$ be a bijection. If both the functions f and the inverse function $f^{-1}: Y \to X$ are continuous then f is called (a) Homeomorphism (b) Isomorphism (c) Homomorphism (d) Epimorphism	K1	CO2
	4	Let $\{X_\alpha\}_{\alpha \in J}$ be an indexed family of topological spaces. Let us take as a basis for a topology on the product space $\prod_{\alpha \in J} X_\alpha$ the collection of all sets of the form $\prod_{\alpha \in J} U_\alpha$ where U_α is open in X_α for each $\alpha \in J$ the topology generated by this basis is called (a) Product topology (b) Discrete topology (c) Box topology (d) Standard topology	K2	CO2
3	5	Let X denote a two point space in the indiscrete topology then X is (a) Continuous (b) Connected (c) Closed (d) Open	K1	CO3
	6	Let X be an ordered set in the order topology. If X is connected then X is a (a) Connected set (b) Continuous set (c) Linear continuum (d) Compact set	K2	CO3
4	7	If every infinite subset of X has a limit point then the space X is said to be (a) Compact (b) Limit point compact (c) Continuous (d) Separable	K1	CO4
	8	If there is some compact subspace C of X that contains a neighborhood of x then the space X is said to be (a) Locally connected at x (b) Locally continuous at x (c) Locally compact atx (d) Locally bounded at x	K2	CO4
5	9	Every well ordered set X is normal in the (a) Order topology (b) Product topology (c) Discrete topology (d) Box topology	K1	CO5
	10	Every locally compact Hausdroff space is (a) Normal (b) Continuous (c) Completely regular (d) Compact	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each x in U there is an element C of $\mathcal{C}$ such that $x \in C \subset U$ then prove that $\mathcal{C}$ is a basis for the topology of X.	K4	CO1
		(OR)		
	11.b.	Let X be an ordered set in the order topology, let Y be a subset of X that is convex in X then show that the order topology on Y is the same as the topology Y inherits as a subspace of X.		
2	12.a.	State and prove the pasting Lemma.	K3	CO2
		(OR)		
	12.b.	Let $f: A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$ where $f_\alpha: A \to X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology then prove that the function f is continuous if and only if each function f_α is continuous.		
3	13.a.	If Y is a subspace of X a separation of Y is a pair of disjoint nonempty sets A and B whose union is Y neither of which contains a limit point of the other, Then prove that the space Y is connected if there exists no separation of Y.	K3	CO3
		(OR)		
	13.b.	Let Y be a subspace of X then prove that Y is compact if and only if every covering of Y by sets open in X contains a finite subcollection covering Y.		
4	14.a.	Suppose that X has a countable basis then show that (i) Every open covering of X contains a countable subcollection covering X? (ii) There exists a countable subset of X that is dense in X.	K4	CO4
		(OR)		
	14.b.	Let X be a topological space. Let one point sets in X be closed then examine (i) X is regular if and only if given a point x of X and a neighborhood U of x there is a neighborhood V of x such that $\bar{V} \subset U$? (ii) X is normal if and only if given a closed set A and an open set U containing A there is an open set V containing A such that $\bar{V} \subset U$.		
5	15.a.	If X is a compact m-manifold then check whether X can be imbedded in $\mathbb{R}^N$ for some positive integer.	K5	CO5
		(OR)		
	15.b.	State and prove Tychonoff theorem.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions
ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Let A be a subset of the topological space X then check whether the following are true (i) $x \in \bar{A}$ if and only if every open set U containing x intersects A. (ii) Suppose the topology of X is given by a basis then $x \in \bar{A}$ if and only if every basis element B containing x intersects A.	K4	CO1
2	17	Let $\bar{d}(a,b) = min\{\lvert a-b\rvert, 1\}$ be the standard bounded metric on $\mathbb{R}$. If x and y are two points of $\mathbb{R}^{\omega}$ define $D(x,y) = sup\left\{\frac{\bar{d}(x_i,y_i)}{i}\right\}$ then prove that D is a metric that induces the product topology on $\mathbb{R}^{\omega}$.	K5	CO2
3	18	Prove that a finite product of connected space is connected.	K5	CO3
4	19	Let X be a space then prove that X is locally compact Hausdroff if and only if there exists a space Y satisfying the following conditions (i) X is a subspace of Y (ii) The set $Y - X$ consists of a single point (iii) Y is a compact Hausdroff space. Also prove if Y and Y' are two spaces satisfying these conditions then there is a homeomorphism of Y with Y' that equals the identity map on X.	K4	CO4
5	20	Prove that every regular space with a countable basis is normal.	K4	CO5

Z-Z-Z END