22MAP102N/22MAP102

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2024
(First Semester)

Branch - **MATHEMATICS**

REAL ANALYSIS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Suppose $f'(x) > 0$ in (a, b) then (a). f is strictly increasing (b). f is strictly decreasing (c). f is oscillating finitely (d). f is oscillating infinitely	K1	CO3
	2	Let f be a differentiable real function defined in (a, b) then f is convex if and only if (a) f is monotonically increasing (b). f' is monotonically decreasing (c). f is monotonically decreasing (d). f' is monotonically increasing	K2	CO1
2	3	If $f \in \Re(\alpha)$ on $[a. b]$ and if $\lvert f(x)\rvert \leq M$ on $[a, b]$ then (a). $\left\lvert\int_a^b f d\alpha\right\rvert \leq M[\alpha(b) - \alpha(a)]$ (b). $\left\lvert\int_a^b f d\alpha\right\rvert \geq M[\alpha(b) - \alpha(a)]$ (c). $\left\lvert\int_a^b f d\alpha\right\rvert \leq M[\alpha(b) + \alpha(a)]$ (d). $\left\lvert\int_a^b f d\alpha\right\rvert \geq M[\alpha(b) + \alpha(a)]$	K1	CO3
	4	The partition P^* is a refinement of P if (a). $P \supset P^*$ (b).$P^* = P$ (c).$P^* \supset P$ (d). $PP^* = 1$	K2	CO1
3	5	If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \to f$ uniformly on E then (a). f is continuous on E (b). f is not continuous on E (c). $\{f_n\}$ is not pointwise converge to f on E (d). $\{f_n\}$ is uniformly continuous on E	K1	CO3
	6	Let $f_n(x) = \frac{x^2}{x^2+(1-nx)^2}$, $0 \leq x \leq 1$, $n = 1,2,3,\ldots$ then which of the following is not true (a). $\lvert f_n(x)\rvert \leq 1$ (b). $\{f_n\}$ is uniformly bounded on $[0,1]$ (c). $\{f_n(x)\}$ is not equicontinuous (d). $\{f_n(x)\}$ is equicontinuous	K2	CO3
4	7	Given a double sequence $\{a_{ij}\}$, $i = 1,2,\ldots j = 1,2,\ldots$.suppose that $\sum_{j=1}^{\infty}\lvert a_{ij}\rvert = b_i$, $i = 1,2,\ldots$ and $\sum b_i$ converges than (a) .$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} > \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$ (b). $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$ (c). $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} < \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$ (d). $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} \neq \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$	K1	CO1
	8	Let $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$ be defined on R' then $e^x$is continuous and differentiable (a). for all x (b). $(e^x)' \neq e^x$ (c). $e^{x+y} \neq e^x e^y$ (d). $e^x$is continuous and not differentiable for all x	K2	CO1
5	9	Let X be a vector space. An operator $P \in L(X)$ is said to be a projection in X if (a) $P^2 = P$ (b) $P^* = P$ (c) $P^{-1} = P$ (d) $P^n = P$	K1	CO1
	10	Suppose X is a vector space and $dim\, X = n$, A set E of a vectors in X spans X if and only if E is (a) Dependent (b).Linear (c) Independent (d) Nilpotent	K2	CO1

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Suppose f is a real differentiable function on $[a,b]$ and suppose $f'(a) < \lambda < f'(b)$ then prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$?	K3	CO2
		(OR)		
	11.b.	If f and g are continuous real functions on $[a,b]$ which are differentiable in (a,b) then prove there is a point $x \in (a,b)$ at which $[f(b) - f(a)]g'(x) = [g(b) - g(a)]f'(x)$		
2	12.a.	Prove that $\underline{\int_a^b} f d\alpha \leq \overline{\int_a^b} f d\alpha$	K2	CO2
		(OR)		
	12.b.	Suppose f is bounded on $[a,b]$, f has only finitely many points of discontinuity on $[a.b]$ and α is continuous at every point at which f is discontinuous then prove that $f \in \Re(\alpha)$?		
3	13.a.	Let α be monotonically increasing on [a, b] for $n = 1,2,3,\dots$ and suppose $f_n \to f$ uniformly on $[a,b]$ then prove that $f \in \Re(\alpha)$ on $[a,b]$ and $\int_a^b f(x)d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$	K3	CO2
		(OR)		
	13.b.	If K is a compact metric space ,if $f_n \in \zeta(K)$ forn $= 1,2,3,\dots$ and if $\{f_n\}$ converges uniformly on K Then prove that $\{f_n\}$ is equicontinuous on K ?		
4	14.a.	Suppose $\sum c_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x^n$ $(-1 < x < 1)$ prove that $\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} c_n$	K3	CO2
		(OR)		
	14.b.	If $x > 0$ and $y > 0$ then Prove that $\int_0^1 t^{x-1}(1-t)^{y-1}dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$?		
5	15.a.	If $A \in L(R^{n+m}, R^n)$ and if A_x is invertible then prove that there corresponds to every $k \in R^m$ a unique $h \in R^n$ such that $(h,k) = 0$?	K2	CO1
		(OR)		
	15.b.	Prove that a linear operator A on a finite- dimensional vector space X is one-to one if and only if the range of A is all of X?		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and Prove Taylor's theorem.	K2	CO1
2	17	If γ' is continuous on $[a,b]$ then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b \lvert\gamma'(t)\rvert\, dt$	K3	CO2
3	18	State and prove Stone-weierstrass theorem.	K2	CO1
4	19	If f is a positive function on $(0,\infty)$ such that (i). $f(x+1) = xf(x)$ (ii). $f(1) = 1$ (iii). $\log f$ is convex then prove that $f(x) = \Gamma(x)$	K3	CO2
5	20	Suppose m, n, r are nonnegative integers, $m \geq r$, $n \geq r$. F is a $\wp'$- mapping of an open set $E \subset R^n$ in to R^m and $F'(x)$ has rank r for every $x \in E$. Fix $a \in E$, Put $A = F'(a)$. Let Y_1 be the range of A and let P be a projection in R^m whose range is Y_1. Let Y_2 be the null space of P then prove that there are open sets U and V in R^n with $a \in U, U \subset E$ and there is a 1-1 $\wp'$ mapping H of V on to U such that $F(H(x)) = Ax + \varphi(Ax)$ $(x \in E)$	K4	CO4

Z-Z-Z END