PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION MAY 2024
(Second Semester)

Branch – MATHEMATICS

PARTIAL DIFFERENTIAL EQUATIONS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)
Answer ALL questions
ALL questions carry EQUAL marks $(10 \times 1 = 10)$

Module No.	Question No.	Question	K Level	CO
1	1	The partial differential equation of $x^2 + y^2 + (z - c)^2 = a^2$ is ___. a) $yp = xq$ b) $yq = xp$ c) $z = pq$ d) $zp = q$	K2	CO2
	2	From the Cauchy's problem if $x_0(\mu)$, $y_0(\mu)$ are Yo functions which, together with their first derivatives are continuous in the interval M defined by ___. a) $\mu_1 = \mu_2 = \mu$ b) $\mu_1 \neq \mu_2 \neq \mu$ c) $\mu_1 < \mu < \mu_2$ d) $\mu_1 > \mu > \mu_2$	K2	CO1
2	3	One dimensional diffusion equation is ___. a) $\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{K}$ b) $\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{K}\frac{\partial \phi}{\partial t}$ c) $\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial t}$ d) $\frac{\partial \phi}{\partial x} = \frac{1}{K}\frac{\partial \phi}{\partial t}$	K1	CO1
	4	F(D, D') is reducible if it can be written as ___. a) $D+D' = 0$ b) $DD' = 0$ c) $D - D' = 0$ d) $D + ab' + b$	K2	CO3
3	5	If $\psi(x, y, z)$ is a solution of Laplace's equation then the family of equipotential surface is___. a) $\psi(x, y, z) = c$ b) $\psi(x, y, z) = \{x, y, z\}$ c) $\psi(x, y, z) = 0$ d) $\psi(x, y, z) = \{0, 1\}$	K1	CO1
	6	At any point which the density of gravitating matter is ρ the potential ψ satisfies Poisson's equation is ___. a) $\nabla^2 \psi = 4\pi\rho$ b) $\nabla^2 \psi = -4\pi\rho$ c) $\nabla^2 \psi = 2\pi\rho$ d) $\nabla^2 \psi = -2\pi\rho$	K2	CO2
4	7	The transverse vibrations of a thin membrane S is bounded by ___. a) the curve Γ in xyz plane b) the curve Γ in xy plane c) the curve Γ at origin d) all points of S	K1	CO3
	8	Which of the following is D'Alembert solution of one dimensional wave equation? a) $\frac{1}{2}\{\eta(x + ct) + \eta(x - ct)\}$ b) $\frac{1}{2}\{\eta(x + ct) - \eta(x - ct)\}$ c) $\frac{1}{2}\{\eta(x - ct) + \eta(x)\}$ d) $\frac{1}{2}\{\eta(x - ct) - \eta(x)\}$	K2	CO4
5	9	One dimensional diffusion equation has solutions of the form ___. a) $\theta(x, t) = \sum_\lambda [c_\lambda \cos(\lambda x) + d_\lambda \sin(\lambda x)]$ b) $\theta(x, t) = \sum_\lambda [c_\lambda \cos(\lambda x) + d_\lambda \sin(\lambda x)] e^{-\lambda^2 \kappa t}$ c) $\theta(x, t) = \sum_\lambda [\cos(\lambda x) + \sin(\lambda x)] \kappa e^{-\lambda}$ d) none of these	K1	CO3
	10	When will be the integral $\frac{1}{2\sqrt{\pi \kappa t}} \int_{-\infty}^{\infty} \phi(\xi) \exp\left\{-\frac{(x-\xi)^2}{4\kappa t}\right\} d\xi$ convergent? a) $t = 0$ b) $t < 0$ c) $t > 0$ d) $t \neq 0$	K2	CO5

SECTION - B (35 Marks)
Answer ALL questions
ALL questions carry EQUAL Marks $(5 \times 7 = 35)$

Module No.	Question No.	Question	K Level	CO
1	11.a.	Show that the equations xp = yq and z(xp+yq) = 2xy are compatible and solve them.	K3	CO2
		(OR)		
	11.b.	Find the complete integral of $p^2x + q^2y = z$.		
2	12.a.	Prove: If the operator F(D, D') is reducible, the order in which the linear factors occur is unimportant.	K4	CO3
		(OR)		
	12.b.	Find a particular integral of the equation $(D^2 - D')z = 2y - x^2$.		
3	13.a.	Derive the necessary condition for an one parameter system of surfaces f(x,y,z)=c to be equipotential surfaces.	K3	CO5
		(OR)		
	13.b.	Explain interior Dirichlet problem.		
4	14.a.	Explain about sound waves in space.	K4	CO3
		(OR)		
	14.b.	Derive the elementary solution of Laplace equation.		
5	15.a.	Discuss the various situations in physics with the occurrence of diffusion equation.	K5	CO2
		(OR)		
	15.b.	How to apply method of the separation of variables to diffusion equation?		

SECTION -C (30 Marks)
Answer ANY THREE questions
ALL questions carry EQUAL Marks $(3 \times 10 = 30)$

Module No.	Question No.	Question	K Level	CO
1	16	Find the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.	K4	CO4
2	17	Solve the equation $r + s - 2t = e^{x+y}$.	K4	CO2
3	18	A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.	K5	CO4
4	19	Find approximate values for the first three eigenvalues of a square membrane of side 2.	K4	CO3
5	20	Find the solution of the equation $\kappa \nabla^2 \theta = \frac{\partial \theta}{\partial x}$ at for an infinite solid whose initial distribution of temperature is given by $\theta(r, 0) = f(r)$ where the function f is prescribed.	K5	CO5