22MAP209N/ 22MAP209

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2024
(Second Semester)

Branch – **MATHEMATICS**

MEASURE THEORY

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Which is a measurable set? (a) Open Set (b) Closed Set (c) Borel Set (d) All the above	K1	CO1
	2	The set E is Lebesgue's measurable if for each set A, m*(A)=_________. m*(A∩E) (b) m*(A∩E) + m*(A∩ E^c) (c) m*(A∩ E^c) (d) m*(A∩E) - m*(A∩ E^c)	K2	CO1
2	3	Let f be bounded and let f and $\lvert f\rvert$ be Riemann integral on $(-\infty,\infty)$ then ________. (a) f is integrable (b) $\int_{-\infty}^{\infty} f\,dx = R\int_{-\infty}^{\infty} f\,dx$ (c) a &b (d) $\int_{-\infty}^{\infty} f\,dx \neq R\int_{-\infty}^{\infty} f\,dx$	K1	CO2
	4	$\int_0^1 \left(\frac{\log x}{1-x}\right)^2 dx =$ _______. (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{3}$	K2	CO2
3	5	A measure μ on R is ____ if when ever $E \in R, F \subseteq E$ *and* $\mu(E) = 0$ then $F \in R$. (a) ring (b) σ-ring (c) σ-finite (d) complete	K1	CO3
	6	Let A, B be subsets of a set C, let $A,B,C \in R$ and let μ be a measure on R , i $\mu(A) = \mu(C) < \infty$ then $\mu(A \cap B) =$ _______. (a) $\mu(A)$ (b) $\mu(B)$ (c) $\mu(C)$ (d) $\mu(A \cup B)$	K2	CO3
4	7	Let $f,g \in L^p(\mu)$ and let a,b be constants, then ___________. (a) $af + bg \in L^p(\mu)$ (b) $af \in L^p(\mu)$ (c) $\lVert af\rVert = \lvert a\rvert.\lVert f\rVert$ (d) all	K2	CO4
	8	$\lVert f+g\rVert_\infty \leq$ _______. (a) $\lVert f\rVert_\infty + \lVert g\rVert_\infty$ (b) $\lVert f\rVert_1 + \lVert g\rVert_\infty$ (c) $\lVert f\rVert_1 + \lVert g\rVert_1$ (d) $\lVert f\rVert_1 - \lVert g\rVert_1$	K2	CO4
5	9	A countable union of sets positive with respect to a signed measure v is a ________. (a) Null set (b) negative set (c) positive set (d) none	K1	CO5
	10	A set v defined on a measurable space $[[X,S]]$ is said to be a____ (a) Signed measure (b) measure (c) Hahn decomposition (d) Jordan decomposition	K1	CO5

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that every integral is measurable.	K3	CO1
		(OR)		
	11.b.	Prove that for any sequence of sets $\{E_i\}$, $m^*\left(\bigcup_{i=1}^{\infty} E_i\right) \le \sum_{i=1}^{\infty} m^*(E_i)$		
2	12.a.	Make use the basic concepts, state and prove Lebesgue's Dominated convergence theorem.	K3	CO2
		(OR)		
	12.b.	Use the logical arguments, Show that $\int_0^{\infty} \frac{\sin t}{e^t - x} dt = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n^2+1}$, $-1 \le x \le 1$		
3	13.a.	Let μ^* be the outer measure on H(R) defined by μ on R then prove that S* contains S(R), the $\sigma-$ring generated by R.	K4	CO3
		(OR)		
	13.b.	Prove that the outer measure μ^* on H(R) defined by μ on R by $\mu^*(E) = \inf[\sum_{n=1}^{\infty} \mu(E_n) : E_n \in R, n = 1,2,3,4,......., E \subseteq \bigcup_{n=1}^{\infty} E_n]$ and the corresponding outer measure defined by $\bar{\mu}$ on S(R) and $\bar{\mu}$ on S* are the same.		
4	14.a.	Analyze the statement of Minkowski's inequality.	K4	CO4
		(OR)		
	14.b.	Analyze that every function convex on an open interval is continuous.		
5	15.a.	Criticize the statement of Jordan decomposition theorem.	K5	CO5
		(OR)		
	15.b.	Justify the statement. let v be a signed measure on the measurable space (X, B)then there is a positive set A and a negative set B such that $X = AUB$ and $A \cap B = \phi$. The pair A,B is said to be Hahn decomposition of X with respect to v. it is unique to the extent that if A_1, B_1 and A_2, B_2 are Hahn decompositions of X with respect to v then $A_1 \Delta A_2$ is a v-null set.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Prove that the outer measure of an interval equals its length.	K5	CO1
2	17	State and prove Fatou's lemma.	K5	CO2
3	18	Analyze the statement. Let μ^* be an outer measure on H® and let S* denote the class of μ^* measurable sets. Then S* is a $\sigma-$ring and μ^* restricted to S* is a complete measure.	K4	CO3
4	19	Analyze the statement ofss Jensen's inequality by providing its proof.	K4	CO4
5	20	Analyze the statement of Radon-Nikodym theorem, by proving it.	K4	CO5

Z-Z-Z END