23MCU414

PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024

(Fourth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

SEQUENCE, SERIES AND TRIGONOMETRY

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 $\lim_{n\to\infty} \frac{1}{n} =$ _______.

(i) 1 (ii) ∞ (iii) 0 (iv) -1

2 $\liminf_{n\to\infty} (-1)^n =$ _______.

(i) -1 (ii) 1 (iii) 0 (iv) n

3 $\sum_{n=1}^{\infty} \frac{x^n}{n}$ is absolutely convergent only for ________.

(i) $-\infty < x < \infty$ (ii) $-1 < x < 1$

(iii) $-2 < x < 2$ (iv) $-1 < x < 0$

4 $e^{i\theta} = cos\theta + isin\theta$ is known as _______.

(i) Euler's formula (ii) Exponential formula

(iii) cosine formula (iv) sine formula

5 $\tan^{-1} x$ lies between ________.

(i) $\pm\frac{\pi}{4}$ (ii) $\pm\frac{\pi}{2}$ (iii) $\pm\pi$ (iv) $\pm 2\pi$

SECTION - B (15 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.

OR

b If $\{s_n\}_{n=1}^{\infty}$ is a sequence of non negative numbers and if $\lim_{n\to\infty} s_n = L$, then show that $L \geq 0$.

7 a If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges, then prove that $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

OR

b Prove that $\lim_{n\to\infty} \frac{2n}{n+4n^{1/2}} = 2$.

8 a If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then prove that $\lim_{n\to\infty} a_n = 0$.

OR

b (i) Show that if $\sum_{n=1}^{\infty} a_n$ converges absolutely then both $\sum_{n=1}^{\infty} p_n$ and $\sum_{n=1}^{\infty} q_n$ converge and

(ii) if $\sum_{n=1}^{\infty} a_n$ converges conditionally, then both $\sum_{n=1}^{\infty} p_n$ and $\sum_{n=1}^{\infty} q_n$ diverge.

Cont ...

9 a Develop $sin^7\theta$ into a series of sines of multiples of θ.

OR

b Solve $\lim_{\theta\to 0}\frac{n\, sin\theta - \sin n\theta}{\theta(cos\theta - sinn\theta)}$.

10 a Calculate the sum of series $cos^2x + cos^2(x+y) + cos^2(x+2y)$ ..... up to n terms.

OR

b State and prove Gregory's series.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a (i) Define monotone sequence.

(ii) Justify that if the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent to L, then $\{s_n\}_{n=1}^{\infty}$ cannot also converge to a limit distinct from L.

OR

b (i) Prove that a nondecreasing sequence which is not bounded above diverges to infinity.

(ii) Show that the sequence $\{(1+\frac{1}{n})^n\}_{n=1}^{\infty}$ is convergent.

12 a If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequence of real numbers, if $\lim_{n\to\infty} s_n = L$ and if $\lim_{n\to\infty} t_n = M$ then, justify that $\lim_{n\to\infty}(s_n + t_n) = L + M$.

OR

b Prove that any bounded sequence of real numbers has a convergent subsequence.

13 a (i) Define class l^2.

(ii) State and prove Schwarz inequality.

OR

b Show that if $\sum_{n=1}^{\infty} a_n$ is a divergent series of positive numbers, then there is a sequence $\{\varepsilon_n\}_{n=1}^{\infty}$ of positive numbers which converges to zero but for which $\sum_{n=1}^{\infty}\varepsilon_n a_n$ still diverges.

14 a (i) If $\tan(x+iy) = u + iv$, prove that $\frac{u}{v} = \frac{\sin 2x}{sinh2y}$.

(ii) Seperate into real and imaginary parts of $tan\, h(1+i)$.

OR

b Express $cos8\theta$ in terms of $sin\theta$.

15 a (i) State and prove the general value of logarithm of $x + iy$.

(ii) Find the sum of the series

$$cosec\theta + cosec\, 2\theta + cosec2^2\theta + \cdots + cosec2^{n-1}\theta.$$

OR

b Find the sum to infinity the series

$$cos\alpha + \frac{1}{2}\cos(\alpha+\beta) + \frac{1.3}{2.4}\cos(\alpha+2\beta) + \cdots.$$

Z-Z-Z END