

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(First Semester)

Branch – MATHEMATICS WITH COMPUTER APPLICATIONS

ORDINARY DIFFERENTIAL EQUATIONS & LAPLACE TRANSFORMS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

- Write the differential equation for the situation, the time rate of change of a population $p(t)$ with constant birth and death rates is proportion to the size of the population?
 - $\frac{dp}{dt} = \frac{k}{p}$
 - $\frac{dp}{dt} = kp$
 - $\frac{dp}{dt} = k\sqrt{p}$
 - $\frac{dp}{dt} = \frac{k}{\sqrt{p}}$
- Which one of the definition of velocity of a possible?
 - $v(t) = f'(t)$
 - $a(t) = f''(t)$
 - $v(t) = f(t)$
 - $v(t) = f''(t)$
- Write the general solution if the characteristic equation $ay'' + by' + c = 0$ has the roots which is real and distinct?
 - $y(x) = (c_1 + c_2x)e^{r_1x} + r_2x$
 - $y(x) = c_1e^{r_1x} + c_2e^{r_2x}$
 - $y(x) = c_1 \cos x + c_2 \sin x$
 - $y(x) = c_1r_1x + c_2r_2x$
- What is $W(\cos x, \sin x)$, where W represents wronskian?
 - 1
 - 0
 - ∞
 - None
- What are the roots of the characteristic equation $r^3 + r^2 = 0$
 - $r_1 = 1, r_2 = 0, r_3 = 0$
 - $r_1 = r_2 = 1, r_3 = 0$
 - $r_1 = r_2 = 0, r_3 = -1$
 - $r_1 = r_2 = 0, r_3 = 1.$
- What is the complementary function $y'' + y = \tan x$?
 - $y_c(x) = c_1 \cos x + c_2 \sin x$
 - $y_c(x) = (c_1 + c_2x)e^x$
 - $y_c(x) = c_1 \cos x - c_2 \sin x$
 - $y_c(x) = c_1 \sin x - c_2 \cos x$
- What is $L(\cos hkt)$?
 - $\frac{s}{s^2+k^2}$
 - $\frac{s}{s^2-k^2}$
 - $\frac{k}{s^2+k^2}$
 - $\frac{k}{s^2-k^2}$
- What is $L(f'(t))$?
 - $SF(S) - f(0)$
 - $S^2F(S) - f'(0)$
 - $SF(S) + f(0)$
 - $S^2F(S) - Sf(0)$
- $L\left(\frac{f(t)}{t}\right) = ?$
 - $\int_0^\infty e^{-st} f(t) dt$
 - $\int_s^\infty F(\sigma) d\sigma$
 - $\int_0^\infty e^{st} f(t) dt$
 - None.
- $L^{-1}[F(s).G(s)] = ?$
 - $f(t) * g(t)$
 - $f(t).g(t)$
 - $\frac{f(t)}{g(t)}$
 - $\frac{F(s)}{G(s)}$

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

- 11.a. Solve the differential equation. $\frac{dy}{dx} - y = \frac{11}{8}e^{-\frac{x}{3}}$.

(OR)

Cont...

- 11.b. Solve the differential equation. $\frac{dy}{dx} = (x + y + 3)^2$
- 12.a. If $y_1(x) = e^{2x}$ and $y_2(x) = e^{-2x}$ then verify the y_1 and y_2 are linearly independent or dependent?
(OR)
- 12.b. Verify that the functions $y_1(x) = e^x$ and $y_2(x) = xe^x$ are solutions of the differential equation $y'' - 2y' + y = 0$.
- 13.a. The roots of the characteristic equation of a certain differential equation are $3, -5, 0, 0, 0, -5, 2 \pm 3i$ and $2 \pm 3i$. Write a general solution of this homogeneous differential equation?
(OR)
- 13.b. Find a particular solution of $y'' - 4y = 2e^{3x}$.
- 14.a. Show that $L(te^{at}) = \frac{1}{(s-a)^2}$.
(OR)
- 14.b. Find the inverse Laplace Transform of $G(s) = \frac{1}{s^2(s-a)}$.
- 15.a. Find $L^{-1}\left[\frac{2s}{(s^2-1)^2}\right]$.
(OR)
- 15.b. Find $L^{-1}\left[\tan^{-1}\left(\frac{1}{s}\right)\right]$.

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

16. Solve the differential equation $2xy \frac{dy}{dx} = 4x^2 + 3y^2$
17. Show that the three solutions $y_1(x) = x$, $y_2(x) = x \ln x$ and $y_3(x) = x^2$ of the third order equation $x^3 y^{(3)} - x^2 y'' + 2xy' - 2y = 0$ are linearly independent on the open interval $x > 0$. Then find the particular solution of the given equation?
18. Solve the initial value problem $y^3 + 3y'' - 10y' = 0$ given that $y(0) = 7, y'(0) = 0, y''(0) = 70$
19. Solve the initial value problem by using Laplace Transforms $x'' - x' - 6x = 0; x(0) = 2, x'(0) = -1$.
20. Find $L[g(t)]$ if $g(t) = \begin{cases} 0 & \text{if } t < 3 \\ t^2 & \text{if } t \leq 3 \end{cases}$

Z-Z-Z

END