18MCU29

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

COMPLEX ANALYSIS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 The limit of a function $f(z)$ exists at a point z_0 is ________.
(i) finite (ii) infinite (iii) unique (iv) not unique

2 A set of points $z = (x, y)$ in the complex plane is said to be an _____ if $x = x(t)$, $y = y(t)$ where $x(t)$ and $y(t)$ are continuous functions of the real parameter t.
(i) arc (ii) interval (iii) mapping (iv) equation

3 If a function f is analytic at a given point, then its _____ of all orders are analytic there too.
(i) neighborhood (ii) derivatives
(iii) antiderivatives (iv) values

4 The function $\frac{z+1}{z^3(z^2+1)}$ has isolated singular points at ______.
(i) $z = 0$ and $z = \pm 1$ (ii) $z = i$ and $z = \pm 1$
(iii) $z = \pm 2$ and $z = \pm i$ (iv) $z = 0$ and $z = \pm i$

5 The polynomial $f(z) = z^3 - 8$ has a zero of order $m =$ _____ at $z_0 = 2$.
(i) 1 (ii) 3 (iii) 2 (iv) 4

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Explain about the stereographic projection.

OR

b Show that the function $f(z) = z^2$ is differentiable everywhere and also find $f'(z)$.

7 a Evaluate the integral $\int_0^{\pi/4} e^{it} dt$.

OR

b Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant then show that $\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}$.

8 a State and prove Liouville's theorem.

OR

b Find the Maclaurin series expansion of the function $f(z) = \sin z$ ($|z| < \infty$).

9 a State and prove Cauchy's Residue theorem.

OR

b Determine the order m of each pole for the function $f(z) = \frac{z^3+2z}{(z-i)^3}$ and find the corresponding residue B.

Cont...

10 a Find the residue of the function $f(z) = \frac{\tan hz}{z^2}$ at $z = \frac{\pi i}{2}$.

OR

b Evaluate the integral $\int_0^\infty \frac{x^2}{x^6+1}\, dx$.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 x 6 = 30)

11 a State and prove the sufficient conditions for differentiability.

OR

b Prove that if $f'(z) = 0$ everywhere in a domain D, then $f(z)$ must be constant throughout D.

12 a Suppose that a function $f(z)$ is continuous on a domain D. If any one of the following statements is true, then so are the others:

(i) $f(z)$has an antiderivative $F(z)$ throughout D;

(ii) the integrals of $f(z)$ along contours lying entirely in D and extending from any fixed point z_1 to any fixed point z_2 all have the same value, namely $\int_{z_2}^{z_1} f(z)dz = F(z)\Big]_{z_1}^{z_2} = F(z_2) - F(z_1)$ where $F(z)$ is the antiderivative in statement (i).

(iii) the integrals of $f(z)$ around closed contours lying entirely in D all have value zero.

OR

b Let f be analytic throughout a closed region R consisting of the points interior to a positively oriented simple closed contour C together with the points on C itself. For any positive number $\in$, the region R can be covered with a finite number of squares and partial squares, indexed by $j = 1,2,\ldots.,n$, such that in each one there is a fixed point z_j for which the inequality

$$\left|\frac{f(z)-f(z_j)}{z-z_j} - f'(z_j)\right| < \in$$

is satisfied by all points other than z_j in that square or partial square.

13 a State and prove Laurent's theorem.

OR

b State and prove maximum modulus principle.

14 a Evaluate the integral $\int_C \frac{dz}{z(z-2)^4}$ where C is the positively oriented circle $|z-2| = 1$.

OR

b An isolated singular point z_o of a function f is a pole of order m if and only if $f(z)$ can be written in the form $f(z) = \frac{\phi(z)}{(z-z_o)^m}$, where $\phi(z)$ is analytic and nonzero at z_o. Moreover, $\underset{z=z_o}{Res} f(z) = \phi(z_o)$ if $m = 1$ and $\underset{z=z_o}{Res} f(z) = \frac{\phi^{(m-1)}(z_o)}{(m-1)!}$ if $m \geq 2$.

15 a Let a function f be analytic at a point z_o. It has a zero of order m at z_o if and only if there is a function g, which is analytic and nonzero at z_o, such that $f(z) = (z-z_o)^m g(z)$.

OR

b State and prove Jordan's lemma.

Z-Z-Z END