PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Fourth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ADVANCED MATHEMATICAL STATISTICS II

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 For stratified random sampling, the mean of the estimate $\overline{y_{st}}$ is_____ .

(i) $\sum_{h=1}^{L} w_h \overline{x_{ha}}$ (ii) $\sum_{h=1}^{L} w_h \overline{Y_h}$

(iii) $\sum_{h=1}^{L} w_{he} \overline{Y_h}$ (iv) $\sum_{h=1}^{L} w_{he} Y_h$

2 Variance of MLE is

(i) $I(\theta)$ (ii) $-I(\theta)$

(iii) $\frac{-1}{I(\theta)}$ (iv) $\frac{1}{I(\theta)}$

3 Power of a test related to

(i) Type I error (ii) Type II error

(iii) Type I error, Type II error (iv) Critical region

4 The ratio between sample variance and with in sample variance follows

(i) F – distribution (ii) χ^2 distribution

(iii) Z –distribution (iv) t – distribution

5 The degrees of freedom for χ^2 is

(i) $(r+1)(c-1)$ (ii) $r-1$

(iii) $c-1$ (iv) $(r-1)(c-1)$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Distinguish between stratified and systematic sampling

OR

b State and prove Rao Blackwell Theorem.

7 a Calculate the M.L.E of parameters α and λ of the distribution $f(x;\alpha;\lambda) = \frac{1}{\Gamma(\lambda)}\left(\frac{\lambda}{\alpha}\right)^{\lambda} e^{-\lambda x/\alpha} x^{\lambda-1}; 0 \le x < \infty$. You may use that for large values of λ, $\psi(\lambda) = \frac{\lambda}{\partial\lambda}\log\Gamma(\lambda) = \log\lambda - \frac{1}{2\lambda}$ and $\psi^1(\lambda) = \frac{1}{\lambda} + \frac{1}{2\lambda^2}$.

OR

b For the double Poisson distribution :

$$p(x) = P(X = x) = \frac{1}{2}\frac{e^{-m_1}.m_1^x}{x!} + \frac{1}{2}\frac{e^{-m_2}.m_2^x}{x!}; x = 0,1,2,.....$$

show that the estimates for m_1 and m_2 by the method of moments are : $\mu_1^1 \pm \sqrt{\mu_2^1 - \mu_1^1 - \mu_1^{1^2}}$

Cont...

8 a Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error and the power of the test.

OR

b Derive the constants of 't' distribution.

9 a An operative claims that he produces 40 articles in an hour. A sample of 10 random hours show the turn out as 43,45,38,37,41,42,44,39,43 and 38. Is the claim of the operative reasonable at 5% significance level?

OR

b A survey of 800 families with four children each revealed the following distribution.

No of boys	0	1	2	3	4
No of girls	4	3	2	1	0
No of families	32	178	290	236	64

Is this result consistent with the hypothesis that male and female births are equally probable?

10 a It is believed that the precision of an instrument is no more than 0.16. Write down the null and alternative hypothesis for testing the belief. Carryout the test at 1% level given 11 measurements of the same subject on the instrument: 2.5, 2.3, 2.4, 2.5, 2.7, 2.5, 2.6, 2.6, 2.7, 2.5

OR

b A die was thrown 498 times. Denoting x to be the number of appearing on the top face it, the observed frequency of x is given below

x	1	2	3	4	5	6
f	69	78	85	82	86	98

What opinion you would form for the accuracy for the die?

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Explain the lottery method and random number table method.

OR

b If t is an unbiased estimator for $\gamma(\theta)$, a function of parameter θ, then prove that $Var(t) \geq \frac{\left\{\frac{d}{d\theta}\gamma(\theta)\right\}^2}{E\left(\frac{\partial}{\partial\theta}\log L\right)^2} = \frac{\{\gamma'(\theta)\}^2}{I(\theta)}$, where $I(\theta)$ is the information on θ, supplied by the sample

12 a The following table gives probabilities and observed frequencies in four classes AB,Ab,aB, and ab in a genetical experiment. Estimate the parameter θ by the method of maximum likely hood and find its standard error.

Class	Probability	Observed frequency
AB	$\frac{1}{4}(2+\theta)$	108
Ab	$\frac{1}{4}(1-\theta)$	27
aB	$\frac{1}{4}(1-\theta)$	30
ab	$\frac{1}{4}\theta$	8

OR

b A random variable X takes the values 0,1,2 with respective probabilities $\frac{\theta}{4N}+\frac{1}{2}\left(1-\frac{\theta}{N}\right)$, $\frac{\theta}{2N}+\frac{\alpha}{2}\left(1-\frac{\theta}{N}\right)$ and $\frac{\theta}{4N}+\frac{1-\alpha}{2}\left(1-\frac{\theta}{N}\right)$, where N is a known number and α,θ are unknown parameters. If 75 independent observations on X yielded the values 0,1,2 with frequencies 27,38,10,respectively, estimate θ and α by the method of moments.

13 a If $x \geq 1$ is the critical region for $H_0 : \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population, $f(x,\theta) = \theta e^{-\theta x}, 0 \leq x < \infty$. Obtain the values of Type I and Type II errors.

OR

b Derive the mode of the F distribution.

14 a The height of six randomly chosen sailors are(in inches): 63,65,68,69,71,and 72. Those of 10 randomly chosen soldiers are 61,62,65,66,69,70,71,72 and 73. Discuss the light that these data throw on the suggestion that sailors are on the average taller than soldiers.

OR

b In a test given to two groups of students drawn from two normal populations, the marks obtained were obtained as follows.

Group A : 18 20 36 50 49 36 34 49 41
Group B : 29 28 26 35 30 44 46

Examine at 5% level, whether the two population have the same variance.

15 a From the table given below, examine whether the colour of son's eyes is associated with that of father's eyes.

		Eye colour in sons	
		Not Light	Light
Eye colour in fathers	Not Light	230	148
	Light	151	471

OR

b On the basis of information noted below, find out whether the new treatment is comparatively superior to the conventional one.

	Favorable	Non favorable	Total
New	140	30	170
Conventional	60	20	80
Total	200	50	N=250

Z-Z-Z END