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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc(SS) DEGREE EXAMINATION MAY 2023
(Fourth Semester)

Branch – **SOFTWARE SYSTEMS**
(Five years Integrated)

TRANSFORMATION TECHNIQUES

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. $L^{-1}\left[\frac{1}{s^{n+1}}\right]$ =-------
   (i) $\frac{t^n}{n}$ (ii) $\frac{t^n}{n!}$ (iii) $\frac{t}{n!}$ (iv) $\frac{t^3}{n!}$

2. The equation $z[n+2](2n - z[n-1]) = n+1$ is---
   i) linear ii) first order
   iii) non linear iv) second order

3. $z(na^n)$ =------
   (i) $\frac{a}{(z-a)^2}$ (ii) $\frac{z}{(z-a)^2}$ (iii) $\frac{az}{(z-a)^2}$ (iv) $\frac{az}{z-a}$

4. If $\mathcal{F}\{f(t)\} = F(\omega)$ and $\mathcal{F}\{g(t)\} = G(\omega)$ then ----
   (i) $\mathcal{F}\{f * g\} = F(\omega)\,G(\omega)$ (ii) $\mathcal{F}\{f * g\} = F(t)\,G(t)$
   (iii) $\mathcal{F}\{fg\} = F(\omega)\,G(\omega)$ (iv) $\mathcal{F}\{F * G) = f(\omega)\,g(\omega)$

5. The linear convolution of the two finite sequences $f[n]$ and $g[n]$ is defined as $h[n] =$
   (i) $\sum_{m=0}^{n} f[m]g[n], n = 0,1,..(N_1 + N_2 - 2)$
   (ii) $\sum_{m=0}^{n} f[m]g[n-m], n = 0,1,..(N_1 + N_2 - 2)$
   (iii) $\sum_{m=0}^{n} f[n-m]g[m], n = 0,1,..(N_1 + N_2 - 2)$
   (iv) $\sum_{m=0}^{n} f[n]g[n-m], n = 0,1,..(N_1 + N_2 - 2)$

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) Find the Laplace transform of $-e^{-t} + \frac{1}{2}(\sin t + \cos t)$.

OR

   b) Find the inverse Laplace transform of $\frac{2s+3}{s^2+6s+13}$.

7. a) Determine $x[4]$ given $2x[k+2] - x[k+1] + x[k] = -k^2$; $x[0] = 1$; $x[1] = 3$.

OR

   b) Design a digital filter based on taking a moving average of the last three values of a sampled signal.

8. a) Find the z transform of the sequence defined by $f(k) = k, k\varepsilon\mathbb{N}$.

OR

   b) Find the sequence whose z transform is $(z) = \frac{2z^2 - z}{(z-5)(z+4)}$.

Cont...

9. a) Find $\mathcal{F}[u(t)e^{-t} + u(t)e^{-2t}]$.

OR

b) Find $\mathcal{F}[\sin at]$.

10. a) Show that the function $\bar{F}(\omega) = T\sum_{n=0}^{N-1} f[n]\, e^{-j\omega nT}$ is periodic with period $\frac{2\pi}{T}$.

OR

b) Verify Rayleigh's theorem for the sequence $f[n] = 5,4$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Solve $\frac{dx}{dt} + x = 9e^{2t}$; $x(0) = 3$ using the Laplace transform.

OR

b) Solve $x'' + 2x' + 2x = e^{-t}$; $x(0) = x'(0) = 0$ using Laplace transform.

12. a) Determine the numerical solution of a difference equation for low pass filter.

OR

b) A computer is fed a signal representing the position of an object as a function of time. Prior to entering the computer, the signal is sampled using an analogue to digital converter. Derive a difference equation and associated block diagram to obtain the acceleration of the object as a function of time.

13. a) The continuous signal $f(t) = \cos\frac{\pi t}{2}$ is sampled at 1 second intervals starting from t=0.

i) Find the Laplace transform of the sampled signal $f^*(t)$.

ii) Show that $F^*(s)$ has an infinity of poles.

iii) Find the z transform of the sampled signal and show that this has just two poles.

OR

b) The sequence $f[k]$ is defined by

$$f[k] = \begin{cases} 0 & k = 0,1,2,3,\ldots \\ 1 & k = 4,5,6,\ldots \end{cases}$$

Write down the sequence $f[k+1]$ and verify that
$\mathcal{Z}\{f[k+1]\} = zF(z) - zf[0]$, $F(z)$ is the transform of $f[k]$.

14. a) Show that the Fourier transform of

$$f(t) = \begin{cases} 3 & -2 \le t \le 2 \\ 0 & otherwise \end{cases}$$

is given by $F(\omega) = \frac{6\sin 2\omega}{\omega}$.

i) Use the first shift theorem to find the Fourier transform of $e^{-jt}f(t)$.

ii) Verify the first shift theorem by obtaining the Fourier transform of $e^{-jt}f(t)$ directly.

OR

b) Find the Fourier transform of $f(t) = \begin{cases} e^{-3t} & ;\ t \ge 0 \\ e^{3t} & ;\ t < 0 \end{cases}$

Deduce the function whose Fourier transform is $G(\omega) = \frac{6}{10+2\omega+\omega^2}$.

15. a) Find the discrete fourier transform of the sequence $f[n] = 1,2,-5,3$.

OR

b) Find the discrete cosine transform $F[k]$ of the sequence $f[n] = 2,4,6$.

Z-Z-Z END