20SSP24

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc(SS) DEGREE EXAMINATION MAY 2023
(Fourth Semester)

Branch – **SOFTWARE SYSTEMS (5 Years Integrated)**

LINEAR ALGEBRA

Time: Three Hours　　　　Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks　　(5 x 1 = 5)

1. A ----- in a matrix A is a location in A that corresponds to a leading 1 in the reduced echelon form of A.
   i) Pivot column　　ii) pivot position
   iii) pivot element　　iv) pivot row

2. Let $p_1(t) = 1, p_2(t) = t$ and $p_3(t) = 4 - t$, Then $\{p_1(t), p_2(t), p_3(t)\}$ is ---.
   i) Linearly dependent　　ii) linearly independent
   iii) basis　　iv) none of these

3. A mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ is said to be ------- if each b in $\mathbb{R}^m$ is the image of at most one x in $\mathbb{R}^n$.
   i) One one　　ii) range　　iii) onto　　iv) domain

4. Let $v = (1, -2, 2, 0)$, The length of v is ---.
   i) 4　　ii) 2　　iii) 1　　iv) 3

5. In the dynamical system $x_{k+1} = Ax_k$, when $A = \begin{bmatrix} 2.0 & 0 \\ 0 & 0.5 \end{bmatrix}$, then the solution x_k is -----.
   i) Unbounded　　ii) bounded　　iii) repeller　　iv) attractor

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks　　(5 x 3 = 15)

6. a) Apply elementary row operations to transform the following matrix first into echelon form and then into reduced echelon form;

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

OR

b) Determine if the following system is consistent;
$x_2 - 4x_3 = 8$; $2x_1 - 3x_2 + 2x_3 = 1$; $5x_1 - 8x_2 + 7x_3 = 1$.

7. a) Determine if the columns of the matrix $A = \begin{pmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{pmatrix}$ are linearly independent.

OR

b) Let $H = \{ (a - 3b, b - a, a, b) : a \text{ and } b \text{ in } \mathbb{R}\}$. Show that H is a subspace of $\mathbb{R}^4$.

8. a) Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}$ and define a transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ by $T(x) = Ax$. Find an x in $\mathbb{R}^2$ whose image under T is b.

OR

b) Define a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}$
Find the images under T of $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $u + v = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

9. a) Let $W = span\{x_1, x_2\}$, $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ .Construct an orthogonal basis $\{v_1, v_2\}$ for W.

OR

9 b) Find a least squares solution of the inconsistent system Ax=b for
$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.

10. a) Let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. An eigenvalue of A is 2. Find a basis for the corresponding eigenspace.

OR

b) Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k, given that $A = PDP^{-1}$, when
$P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Determine if the following homogeneous system has a nontrivial solution. Describe the solution set;
$3x_1 + 5x_2 - 4x_3 = 0\,;\ -3x_1 - 2x_2 + 4x_3 = 0\,; 6x_1 + x_2 - 8x_3 = 0.$

OR

b) Let $A = \begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Is the equation Ax=b consistent for all possible values of b_1, b_2, b_3?

12 a) Given v_1 and v_2 in a vector space V, let $H = span(v_1, v_2\}$. Show that H is a subspace of V.

OR

b) Find a spanning set for the null space of the matrix
$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$.

13. a) Let $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$. Show that T is a one-to-one linear transformation. Does T map $\mathbb{R}^2$ onto $\mathbb{R}^3$.

OR

b) Using the standard basis, find the 4 × 4 matrix P that represents a cyclic permutation T from $x = (x_1, x_2, x_3, x_4)$ to $T(x) = (x_4, x_1, x_2, x_3)$. Find the matrix for T^2. What is the triple shift $T^3(x)$ and why is $T^3 = T^{-1}$? Find two real independent eigenvectors of P. What are all the eigenvalues of P?

14. a) Find the least squares solution of Ax=b for $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$.

OR

b) Find a QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

15. a) Orthogonally diagonalize the matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

OR

b) Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.

Z-Z-Z END