22SSP210

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc(SS) DEGREE EXAMINATION MAY 2023
(Second Semester)

Branch – **SOFTWARE SYSTEMS**
(Five Years Integrated)

DISCRETE STRUCTURES AND APPLIED GRAPH THEORY

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 The logical expression of "You can access the internet from campus only if you are a computer science major or you are not a freshman" is_____ .
(i) $a \rightarrow (c \vee \neg f)$ (ii) $(a \vee c) \rightarrow (\neg f)$
(iii) $a \rightarrow (c \vee f)$ (iv) $a \rightarrow (c \wedge \neg f)$

2 If $f(x) = 2x + 3$ and $g(x) = 3x + 2$ then $g \circ f =$_____ .
(i) $6x + 7$ (ii) $6x - 7$
(iii) $6x + 11$ (iv) $6x - 11$

3 A digraph is called _____ graph, whose underlying graph is a complete graph.
(i) simple (ii) bipartite
(iii) regular (iv) tournament

4 A _____ is an edge-cut consisting of a single edge .
(i) cut-edge (ii) edge-cut
(iii) vertex-cut (iv) cut-vertex

5 If G is an Eulerian graph, then the degree of every vertex is _____ .
(i) odd (ii) equal
(iii) even (iv) unequal

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Show that $\neg(p \vee (p \wedge \neg q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

OR

b Use mathematical induction to show that
$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ for all nonnegative integers n.

7 a Let R be the relation on the set of people such that xRy if x and y are people and x is older than y. Show that R is not a partial ordering.

OR

b Draw the Hasse diagram of the poset $(\{2,4,5,10,12,20,25\},/)$ and which elements of this poset are maximal, and which are minimal?

8 a Prove that a closed trail can be decomposed into edge-disjoint cycles.

OR

b Give the application of edge attributes in Maximum-Flow Problem.

Cont...

9 a Let e be any edge of a k-connected graph G, for $k \geq 3$. Then prove that edge-deletion subgraph $G - e$ is $(k-1)$-connected.

OR

b State and prove the Whitney Synthesis theorem.

10 a Bring out the Eulerian tour algorithm.

OR

b State the three rules for showing that a graph is not a Hamiltonian.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Show that the premises "A student in this class has not read the book", and "Everyone in this class passed the first class", imply the conclusion "Someone who passed the first exam has not read the book".

OR

b Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

12 a Prove that the transitive closure of a relation R equals the connectivity relation R^*.

OR

b Prove that the congruence modulo m is an equivalence relation, where $m > 1$.

13 a Define Eccentricity, Diameter, radius and a central vertex of a graph with example.

OR

b Prove that a graph is bipartite if and only if it has no cycles of odd length.

14 a State and prove the Whitney's 2-connected characterization theorem.

OR

b Prove that the Harary graph $H_{k,n}$ is k-connected, when $k = 2r$.

15 a Let G be a simple n-vertex graph, where $n \geq 3$, such that $\deg(x) + \deg(y) \geq n$ for each pair of non-adjacent vertices x and y. Then prove that G is Hamiltonian.

OR

b Explain the three commonly encountered variations of the TSP that can be transformed to a standard TSP.

Z-Z-Z END